Algebra 2 CP

Midterm Review Chapter 4

Name Key

Add or subtract the following polynomials. (Distribute if necessary, and then combine like terms)

1. $(2x-2y)+(5x+2y)$

$7x$

2. $(5x-7)-(7x-6)$

$5x-7-7x+6$

$-2x-1$

3. $(-3x^3+12x^2-7x)-(9x^2+2x-7)$

$-3x^3+12x^2-7x-9x^2-2x+7$

$-3x^3+3x^2-9x+7$

Multiply the following polynomials. Use distribution or FOIL

4. $-5x^2y^4(7x^5y+3x^2y^3-x^6y^4)$

$-35x^7y^5-15x^4y^7+5x^8y^8$

5. $(x+2)(x-3)$

$x^2-3x+2x-6$

x^2-x-6

6. $(4x+5y)(2x-7y)$

$8x^2-28xy+10xy-35y^2$

$8x^2-18xy-35y^2$

7. $(x+3)^2$ $(x+3)$

x^2+6x+9

8. $(x-4)(x^2-4x+1)$

$x^3-4x^2+x-4x^2+16x-4$

$x^3-8x^2+17x-4$

9. $2(x-3)(x-4)$

$2(x^2-7x+12)$

$2x^2-14x+24$

True or false?

10. $(a+b)^3=a^3+b^3$

$(a+b)(a+b)(a+b)$

$(a^2+2ab+b^2)(a+b)$

$a^3+a^2b+2a^2b+2ab^2+ab^2+b^3$

False!

11. $(a-b)(a^2+ab+b^2)=a^3-b^3$

$a^3+a^2b+ab^2-a^2b-ab^2-b^3$

a^3-b^3 True

12. $(a+b)(a^2-ab+b^2)=a^3+b^3$

$a^3-a^2b+ab^2+a^2b-ab^2+b^3$

a^3+b^3 True

Factor the following polynomials. Look for the greatest common factor, difference of squares, trinomials, and sum/difference of cubes

13. $x^2-3x-10$

$(x-5)(x+2)$

14. $2y^2-50$

$2(y^2-25)$

$2(y+5)(y-5)$

15. $3x^2-18x-24$

$3(x^2-6x-8)$

16. $4a^2 + a - 3$

$(4a-3)(a+1)$

17. $x^3 + 3x^2 - 70x$

$x(x^2+3x-70)$

$x(x+10)(x-7)$

18. $m^3 - 1$ * Formula

$(m-1)(m^2+m+1)$

19. $27r^4y^9 + 18r^5y^6 - 72r^3y^3$

$9r^3y^3(3ry^6+2r^2y^3-8)$

20. $81x^4 - 16$

$(9x^2+4)(9x^2-4)$

$(9x^2+4)(3x+2)(3x-2)$

21. $45x^2 - 80y^2$

$5(9x^2-16y^2)$

$5(3x+4y)(3x-4y)$

22. $8a^3 - 27$ * Formula

$(2a-3)(4a^2+6a+9)$

23. $c^2 + 6c - 7$

$(c+7)(c-1)$

24. $6x^2 - 3x - 18$

$3(2x^2-x-6)$

$3(2x+3)(x-2)$

Solve by <u>factoring</u>

25. $x^2 - 2x = 15$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=5, -3$

26. $4c^2 = 20c$

$4c^2-20c=0$

$4c(c-5)=0$

$c=0, 5$

27. $3x^2 + 4x - 15 = 0$

$(3x-5)(x+3)=0$

$x=5/3, -3$

Simplify

28. $(7-8i)+(-12-4i)$

$-5-12i$

29. $(10-4i)-(7-3i)$

$10-4i-7+3i$

$3-i$

30. $(-3+5i)+(18-7i)$

$15-2i$

31. $(2+i)\cdot(3-5i)$

$6-10i+3i-5i^2$

$6-10i+3i+5$

$\boxed{11-7i}$

32. $(7-6i)\cdot(2-3i)$

$14-21i-12i+18i^2$

$14-33i-18$

$\boxed{-4-33i}$

33. $(3+4i)\cdot(3-4i)$

$9-12i+12i-16i^2$

$9+16$

$\boxed{25}$

34. $\frac{8-6i}{3i}\cdot\frac{3i}{3i}$

$\frac{24i-18i^2}{9i^2} = \frac{18+24i}{-9}$

$= \boxed{-2-\frac{8}{3}i}$

35. $\frac{3i}{4+2i}\cdot\frac{(4-2i)}{(4-2i)}$

$\frac{12i-6i^2}{16-8i+8i-4i^2} = \frac{12i+6}{16+4} = \frac{6+12i}{20}$

$= \boxed{\frac{3}{10}+\frac{3}{5}i}$

Fill in the blanks to complete the square.

36. $x^2-14x+\underline{49}=(x\underline{-}\ \underline{7}\ )^2$

$(-\frac{14}{2})^2=(-7)^2=49$

37. $x^2+9x+\underline{81/4}=(x\underline{+}\ \underline{9/2}\ )^2$

$(\frac{9}{2})^2=\frac{81}{4}$

38. $m^2-\frac{2}{3}m+\underline{1/9}=(m\underline{-}\ \underline{1/3}\ )^2$

$(-\frac{2}{3}/2)^2=(\frac{-2}{3}\cdot\frac{1}{2})^2=(-\frac{1}{3})^2=\frac{1}{9}$

Solve by <u>completing the square</u>

39. $x^2-2x=15$

$(-2/2)^2=1$

$x^2-2x+1=15+1$

$\sqrt{(x-1)^2}=\sqrt{16}$

$x-1=\pm4$

$\boxed{x=5,-3}$

40. $4x^2-8x-12=0$

$4(x^2-2x+\underline{1}\ )=12+\underline{4\cdot1}$

$4(x-1)^2=16$

$(x-1)^2=4$

$x-1=\pm2$

$x=1\pm2$ $\boxed{x=3,-1}$

41. $x^2-7x+5=0$

$x^2-7x+\underline{\frac{49}{4}}=-5+\underline{\frac{49}{4}}$

$(x-7/2)^2=29/4$

$x-7/2=\pm\sqrt{29}/2$

$\boxed{x=\frac{7}{2}\pm\frac{\sqrt{29}}{2}}$

Write each quadratic function in vertex form by COMPLETING THE SQUARE. Identify the vertex, axis of symmetry and direction of opening.

42. $y=x^2+10x+20$

$y-20+\underline{25}=x^2+10x+\underline{25}$

$y+5=(x+5)^2$

$\boxed{y=(x+5)^2-5}$

$\boxed{\begin{array}{l}V:(-5,-5)\\ AoS:\ x=-5\end{array}}$ Opens Up

43. $y=2x^2+16x+29$

$y-29+\underline{2\cdot16}=2(x^2+8x+\underline{16}\ )$

$y+3=2(x+4)^2$

$\boxed{y=2(x+4)^2-3}$

$\boxed{\begin{array}{l}V:\ (-4,-3)\\ AoS:\ x=-4\end{array}}$ Opens Up

Use FOIL to write a quadratic equation with the following roots.

44. 3 and 5 $x = 3 \quad x = 5$

$f(x) = (x-3)(x-5)$

$f(x) = x^2 - 8x + 15$

45. -4*i* and 2 $x = -4i \quad x = 4i \quad x = 2$

$(x+4i)(x-4i)(x-2)$

$(x^2 - 4xi + 4xi - 16i^2)(x-2)$

$(x^2 + 16)(x-2)$

$x^3 - 2x^2 + 16x - 32$

46. $-\frac{2}{5}$ and $\frac{2}{7}$ $x = -2/5 \quad x = 2/7$

$5x = -2 \qquad 7x = 2$

$(5x+2)(7x-2)$

$35x^2 + 4x - 4$

Find the value of the disciminant and determine the nature of the roots. $b^2 - 4ac$

Choices: 2 real roots, 1 real root, 2 complex roots

47. $4x^2 + 16x + 15 = 0$

$(16)^2 - 4(4)(15)$

$256 - 240 = 16$

Disc = 16

Nature of roots 2 Real

48. $x^2 = -10x - 25$

$x^2 + 10x + 25$

$(10)^2 - 4(1)(25)$

$100 - 100 = 0$

Disc = 0

Nature of roots 1 Real

49. $x^2 + 4x + 53 = 0$

$(4)^2 - 4(1)(53)$

$16 - 212 = -196$

Disc = -196

Nature of roots 2 Complex

Solve using the quadratic formula

50. $3x^2 - 5x + 9 = 0$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{-83}}{6}$$

$$x = \frac{5 \pm i\sqrt{83}}{6}$$

51. $x^2 = 4x - 15 \qquad x^2 - 4x + 15 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(15)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-44}}{2}$$

$$= \frac{4 \pm 2i\sqrt{11}}{2} = 2 \pm i\sqrt{11}$$

Calculate the Axis of Symmetry and Vertex by hand, then graph the function. Use the formula: $x = \frac{-b}{2a}$

52. $y = -x^2 + 8x + 3$

$x = -\frac{8}{2(-1)}$

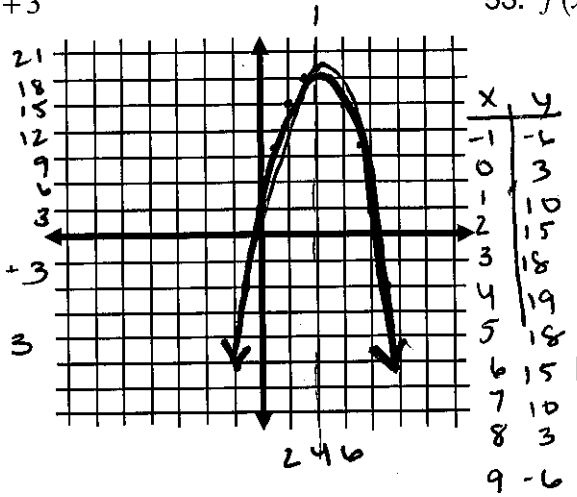
$x = 4$

V: $-(4)^2 + 8(4) + 3$

$= -16 + 32 + 3$

$= 19$

(4, 19)



x	y
-1	-6
0	3
1	10
2	15
3	18
4	19
5	18
6	15
7	10
8	3
9	-6

53. $f(x) = 2x^2 + 2x + 5$

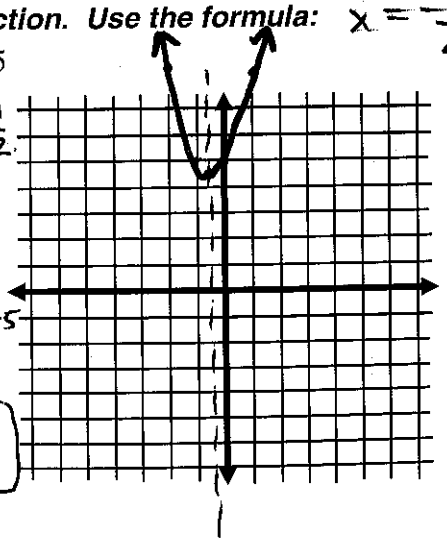
$x = \frac{-2}{2(2)} = -\frac{1}{2}$

$x = -\frac{1}{2}$

$2(-\frac{1}{2})^2 + 2(-\frac{1}{2}) + 5$

$= 4.5$

$(-\frac{1}{2}, 9/2)$



Define a variable, write an equation and solve.

100×30 = original

54. A sports team sells about 100 coupon books for $30 each during its annual fund-raiser. They estimate that for each $0.50 decrease in the price, they will sell about 10 more coupon books. How much should they charge for each book in order to maximize the income from their sales? What is the maximum monthly income the team can expect to make from these items?

x = # price decreases

Income = (# of books)(cost of books)

$= (100+10x)(30-.5x)$

$= 3000 - 50x + 300x - 5x^2$

$= -5x^2 + 250x + 3000$

AoS: $x = \frac{-250}{2(-5)} = 25$

Vertex: $-5(25)^2 + 250(25) + 3000 = 6125$

AoS: 25

Vertex: (25, 6125)

(25, 6125)

Cost = $30 - .5x = 30 - .5(25) = \$17.50$

$17.50 per book, Inc = $6125

55. The sum of two numbers is -1, and their product is -6. Write a quadratic equation to find the two numbers, then solve.

$x + y = -1$ $\quad y = -x - 1$

$x \cdot y = -6$

$x(-x-1) = -6$

$-x^2 - x = -6$

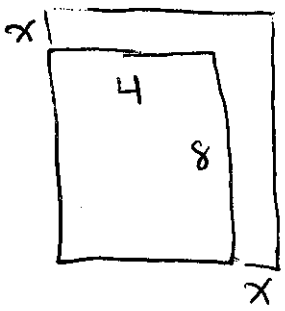
$x^2 + x - 6 = 0$

equation

$(x+3)(x-2) = 0$

$x = -3, 2$

56. A rectangular picture frame measures 8 cm by 4 cm. You want to triple the frame's area by adding the same distance *x* to the length and the width. Write and solve an equation to find the value of *x*. What are the new dimensions of the picture frame?



Original Area = $4 \cdot 8 = 32 cm^2$

New Tripled Area = $32 \cdot 3 = 96 cm^2$

$96 = (4+x)(8+x)$

$96 = 32 + 4x + 8x + x^2$

$x^2 + 12x - 64 = 0$

$(x+16)(x-4) = 0$

$x = -16, 4$

New Dimensions

$4 + 4 = 8$ cm

$8 + 4$ cm $= 12$ cm

Algebra 2 CP

Midterm Review Chapter 5

Name Key

Simplify. Answer all questions using positive exponents!

1. $x^{10} \cdot x^{10}$

x^{20}

2. $\frac{x^{15}}{x^{10}}$

x^5

3. $(y^4)^4$

y^{16}

4. $(2a^2b^3)^4$

$2^4a^8b^{12}$

$16a^8b^{12}$

5. $4j(2j^{-2}k^2)(3j^3k^{-7})$

$24j^2k^{-5} = \frac{24j^2}{k^5}$

6. $(3a^0b^2)(2a^{-3}b^2)^2$

$3b^2 2^2 a^{-6} b^4$

$\frac{12b^6}{a^6}$

7. 2^{-3}

$\frac{1}{2^3} = \frac{1}{8}$

8. $\frac{1}{5^{-2}}$ 5

$5^2 = 25$

9. $(m^2n^4)^{-2}$

$m^{-4}n^{-8} = \frac{1}{m^4n^8}$

10. $\frac{3(a^2b)^4}{(3ab)^3}$ $= \frac{3a^8b^4}{27a^3b^3}$

$\frac{a^5b}{9}$

11. $\frac{4^{-2}x^{-5}y^{-1}}{6y^{-2}x^{-3}}$

$\frac{y^2x^3}{6\cdot 4^2x^5y^1} = \frac{y}{96x^2}$

12. $\frac{1}{x^0 + y^0}$

$\frac{1}{1+1} = \frac{1}{2}$

Divide the following polynomials using synthetic division

13. $(2x^3 - 5x + 40) \div (x+3)$

$$\begin{array}{r|rrrr} -3 & 2 & 0 & -5 & 40 \\ & \downarrow & -6 & 18 & -39 \\ \hline & 2 & -6 & 13 & 1 \end{array}$$

$2x^2 - 6x + 13 + \frac{1}{x+3}$

14. $(x^2 - 4x + 7) \div (x-2)$

$$\begin{array}{r|rrr} 2 & 1 & -4 & 7 \\ & \downarrow & 2 & -4 \\ \hline & 1 & -2 & 3 \end{array}$$

$x - 2 + \frac{3}{x-2}$

Divide the following polynomials using Long Division

15. $(6x^3 + x^2 + x) \div (2x+1)$

$$\begin{array}{r} 3x^2 - x + 1 \\ 2x+1 \overline{)\,6x^3 + x^2 + x + 0} \\ -(6x^3 + 3x^2) \\ \hline -2x^2 + x + 0 \\ -(-2x^2 - x) \\ \hline 2x + 0 \\ -(2x + 1) \\ \hline -1 \end{array}$$

$3x^2 - x + 1 - \frac{1}{2x+1}$

16. $(4x^2 - 2x + 6) \div (2x-3)$

$$\begin{array}{r} 2x + 2 \\ 2x-3 \overline{)\,4x^2 - 2x + 6} \\ -(4x^2 - 6x) \\ \hline 4x + 6 \\ -(4x - 6) \\ \hline 12 \end{array}$$

$2x + 2 + \frac{12}{2x-3}$

Use direct substitution to find $f(2)$ and $f(-1)$ for each function

17. $f(x)=x^3-3x^2+x-2$

$f(2)=(2)^3-3(2)^2+(2)-2$
$=8-12+2-2=$ **-4**

$f(-1)=(-1)^3-3(-1)^2+(-1)-2$
$-1-3-1-2=$ **-7**

18. $f(x)=x^5-7x^3-4x+10$

$f(2)=(2)^5-7(2)^3-4(2)+10$
$=32-56-8+10=$ **-22**

$f(-1)=(-1)^5-7(-1)^3-4(-1)+10$
$=-1+7+4+10=$ **20**

Find $p(-1)$ and $p(2)$ for each function using synthetic substitution.

19. $p(x)=2x^2-4x+1$

-1 | 2 -4 1
$\downarrow$ -2 6
2 -6 7

$p(-1)=7$

2 | 2 -4 1
$\downarrow$ 4 0
2 0 1

$p(2)=1$

20. $p(x)=-2x^3+5x+3$

-1 | -2 0 5 3
$\downarrow$ 2 -2 -3
-2 2 3 0

$p(-1)=0$

2 | -2 0 5 3
$\downarrow$ -4 -8 -6
-2 -4 -3 -3

$p(2)=-3$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

21. x^3+3x^2-6x-8; $x-2$

2 | 1 3 -6 -8
$\downarrow$ 2 10 8
1 5 4 0 ✓

$(x-2)(x^2+5x+4)$

$(x-2)(x+4)(x+1)$

22. $4x^3-12x^2-x+3$; $x-3$

3 | 4 -12 -1 3
$\downarrow$ 12 0 -3
4 0 -1 0 ✓

$(x-3)(4x^2-1)$

$(x-3)(2x+1)(2x-1)$

Solve each equation. State the number and types of roots.

23. $-9x-15=0$

$-3(3x+5)=0$

$x=-5/3$
One Real

24. $x^5=81x$ $\quad x^5-81x=0$

$x(x^4-81)=0$
$x(x^2+9)(x^2-9)=0$
$x(x^2+9)(x+3)(x-3)=0$

$x=0,\pm 3i,-3,3$ 3 real, 2 imag

25. $x^4-5x^2+4=0$

$(x^2-1)(x^2-4)=0$
$(x+1)(x-1)(x+2)(x-2)=0$

$x=-1,1,-2,2$ 4 real

Use Descarte's Rule of Signs. State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

26. $f(x)=4x^3-2x^2+x+3$ $\quad + - + +$ **2 or 0**

$f(-x)=4(-x)^3-2(-x)^2+(-x)+3$
$=-4x^3-2x^2-x+3$ $\quad - - - +$ **1**

+	−	i	T
2	1	0	3
0	1	2	3

27. $f(x)=2x^4-6x^3+6x^2+24x-40$ $\quad + - + + -$ **3 or 1**

$f(-x)=2(-x)^4-6(-x)^3+6(-x)^2+24(-x)-40$
$=2x^4+6x^3+6x^2-24x-40$
$+ + + - -$ **1**

+	−	i	T
3	1	0	4

List all the possible rational zeros of each function.

28. $p(x) = 2x^4 - 5x^3 + 8x^2 + 3x - 5$ p

$\frac{p}{q} = \pm \frac{1, 5}{1, 2}$

$\pm 1, 1/2, 5, 5/2$

29. $g(x) = x^5 - 7x^4 + 3x^2 + x - 20$

$p/q = \pm \frac{1, 2, 4, 5, 10, 20}{1}$

$\pm 1, 2, 4, 5, 10, 20$

Find all of the zeros of each function. synthetic

30. $g(x) = x^3 - 3x^2 - 4x + 12$

-2	1	-3	-4	12
	↓	-2	10	-12
	1	-5	6	0 ✓

$(x+2)(x^2 - 5x + 6)$

$(x+2)(x-2)(x-3)$

$x = -2, 2, -3$

31. $f(x) = 3x^3 - 9x^2 - 10x - 8$

4	3	-9	-10	-8
	↓	12	12	8
	3	3	2	0 ✓

$(x-4)(3x^2 + 3x + 2) = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(3)(2)}}{6}$

$= \frac{-3 \pm \sqrt{-15}}{6} = -\frac{1}{2} \pm \frac{i\sqrt{15}}{6}$

$x = 4, \; -\frac{1}{2} \pm \frac{i\sqrt{15}}{6}$

Find the zeros, relative minimum(s) and relative maximum(s) and graph the following. Your answer must be exact!

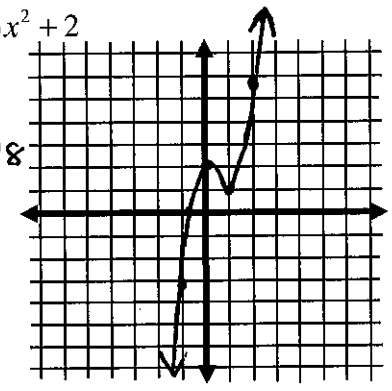
32. $f(x) = 2x^3 - 3x^2 + 2$

-2	-26
-1	-3
0	2
1	1
2	6
3	29

Zeros
x = -.678

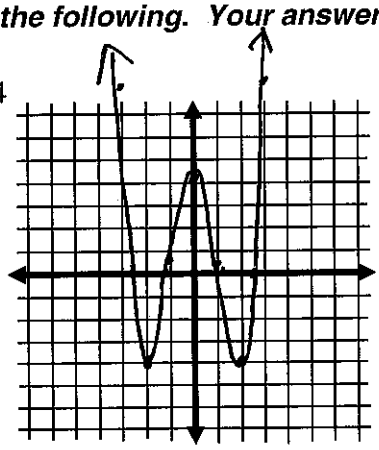
Max
(0, 2)

Min
(1, 1)



33. $f(x) = 0.5x^4 - 4x^2 + 4$

-3	8.5
-2	-4
-1	-.5
0	4
1	.5
2	-4
3	8.5



Zeros
-2.6, 2.6, -1.1, 1.1

Max
(0, 4)

Min
(-2, -4)
(2, -4)

Describe the end behavior for each of the following by completing the statement:

as $x \to -\infty$, $f(x) \to$ _____ ***and*** *as* $x \to \infty$, $f(x) \to$ _____

34. $f(x) = -x^3 - 4x^2 + 5$

As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to \infty$

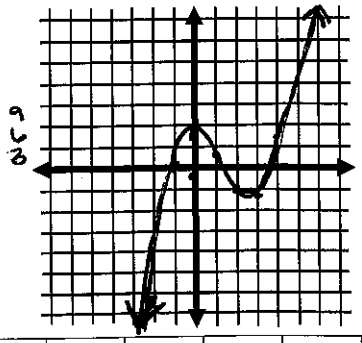
35. $g(x) = 4x^4 - 3x^3 + x^2 + 1$

As $x \to -\infty$, $g(x) \to \infty$

As $x \to \infty$, $g(x) \to \infty$

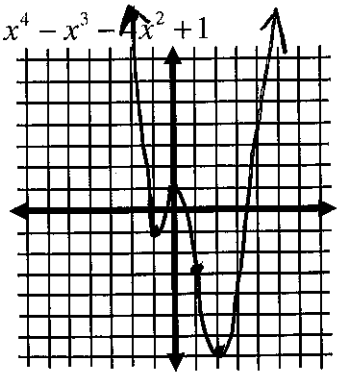
Graph the following polynomials, be sure to include the relative maxima and minima.

36. $f(x) = x^3 - 4x^2 + 5$



x	-3	-2	-1	0	1	2	3
y	-58	-19	0	5	2	-3	-4

37. $g(x) = x^4 - x^3 - 4x^2 + 1$



x	-3	-2	-1	0	1	2	3
y	73	9	-1	1	-3	-7	19

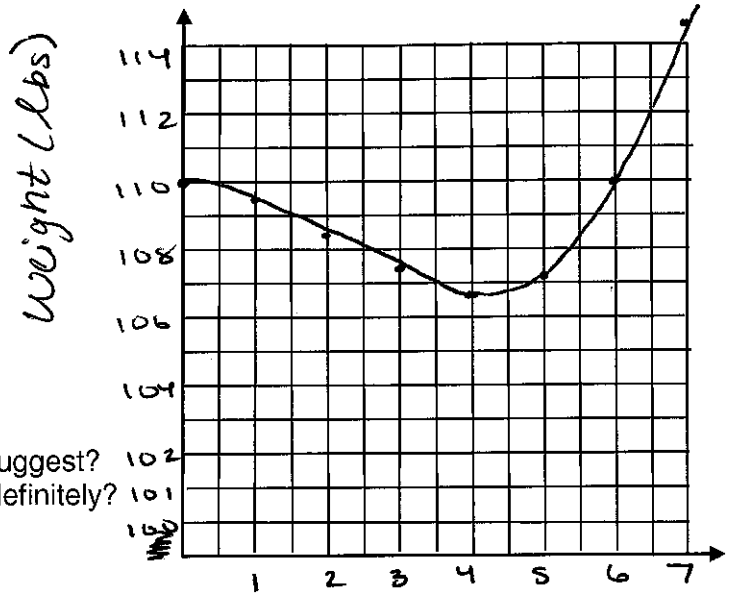
38. The weight *w*, in pounds, of a patient during a 7-week illness is modeled by the cubic equation: $w(n) = 0.1n^3 - 0.6n^2 + 110$, where *n* is the number of weeks since the patient became ill.

a. Graph the equation, and describe the turning points of the graph and its end behavior.

Min @ (4, 106.8)

As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to \infty$



b. What trends in the patient's weight does the graph suggest? Is it reasonable to assume the trend will continue indefinitely?

The patient's weight hits a low at week 4 and then increases quickly. Not reasonable because a person cannot increase forever in weight.

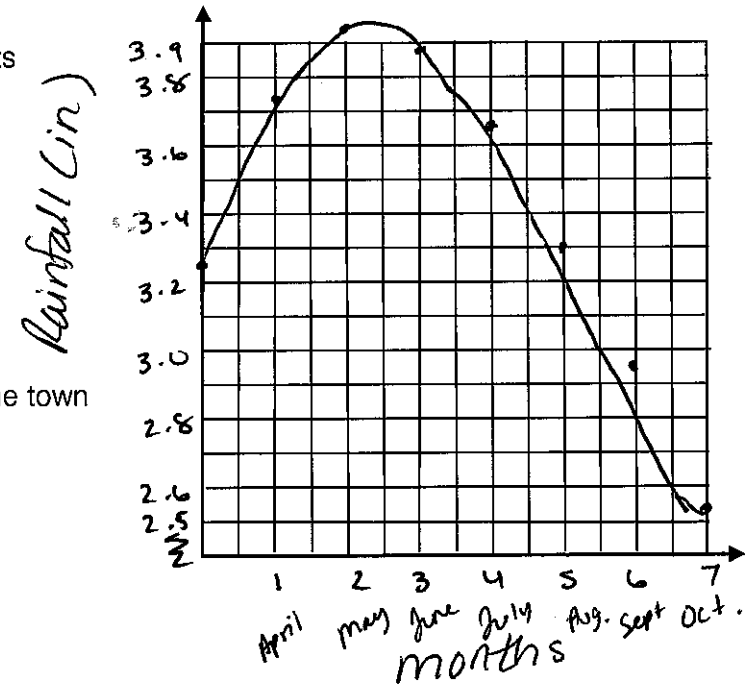
39. The rainfall *r*, in inches per month, during a 7-month period is modeled by the equation: $r(m) = 0.01m^3 - 0.18m^2 + 0.67m + 3.23$, where *m* is the number of months after March 1.

a. Graph the equation, and describe the turning points of the graph and its end behavior.

Max @ (2.3, 3.9)

As $x \to -\infty$, $f(x) \to -\infty$

As $x \to \infty$, $f(x) \to \infty$



b. What trends in the amount of rainfall received by the town does the graph suggest?

The rainfall increases through about May and then decreases through to October.

Algebra 2 CP

Midterm Review Chapter 6

Name Key

Simplify

1. $-\sqrt{121}$

-11

2. $\sqrt[4]{16x^4}$ $\sqrt[4]{2^4 x^4}$

$2x$

3. $\sqrt[3]{64}$

$\sqrt[3]{4^3} = 4$

4. $\sqrt{16x^2y^4}$

$4xy^2$

5. $\sqrt{6x^2y}\cdot\sqrt{6x^3y^7}$

$\sqrt{36x^5y^8}$

$6x^2y^4\sqrt{x}$

6. $\sqrt[3]{4a^2b^5}\cdot\sqrt[3]{12a^2b}$

$\sqrt[3]{2^4\cdot 3\cdot a^4\cdot b^6}$

$2ab^2\sqrt[3]{2\cdot 3\cdot a}$

$2ab^2\sqrt[3]{6a}$

7. $5\sqrt{12}$ $= 5\sqrt{4\cdot 3}$

$10\sqrt{3}$

8. $(6\sqrt{3})(2\sqrt{6})$

$12\sqrt{18} = 12\sqrt{9\cdot 2}$

$= 36\sqrt{2}$

9. $\frac{\sqrt{15}}{\sqrt{5}}$

$\sqrt{\frac{15}{5}} = \sqrt{\frac{3}{1}} = \sqrt{3}$

10. $6\sqrt{2}+3\sqrt{2}-7$

$9\sqrt{2}-7$

11. $3\sqrt{16}-4\sqrt{75}+\sqrt{3}$

$12 - 20\sqrt{3} + \sqrt{3}$

$12-19\sqrt{3}$

12. $(4+\sqrt{6})(4-\sqrt{6})$

$16-4\sqrt{6}+4\sqrt{6}-6$

$16-6 = 10$

Rationalize the denominator.

13. $\frac{2}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$

$\frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

14. $\frac{5}{\sqrt[3]{2}}$ $\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$

$\frac{5\sqrt[3]{2^2}}{2}$ or $\frac{5\cdot 2^{2/3}}{2}$

15. $\frac{3}{1+\sqrt{2}}$ $\frac{(1-\sqrt{2})}{(1-\sqrt{2})}$

$\frac{3-3\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-2} = \frac{3-3\sqrt{2}}{-1}$

$= -3+3\sqrt{2}$

Express the following using rational exponents.

16. $\sqrt[4]{15}$

$15^{1/4}$

17. $\sqrt[3]{x^2}$

$x^{2/3}$

18. $\sqrt[7]{3x^2y^4}$

$3^{1/7}x^{2/7}y^{4/7}$

Express the following in simplest radical form. No decimal answers.

19. $5r^{\frac{1}{6}}s^{\frac{5}{6}}$

$5\sqrt[6]{r^1 s^5}$

20. $x^{\frac{4}{3}}y^{\frac{5}{2}}$ $= x^{8/6}y^{15/6}$

$\sqrt[6]{x^8 y^{15}}$

$xy^2\sqrt[6]{x^2y^3}$

21. $\sqrt[6]{36}$ $\sqrt[6]{6^2} = 6^{2/6}$

$= 6^{1/3}$

$= \sqrt[3]{6}$

22. $4^{\frac{2}{3}}$

$(2^2)^{2/3} = 2^{4/3} = \sqrt[3]{2^4}$

$= 2\sqrt[3]{2}$

23. $\left(5^{\frac{4}{7}}\right)^{\frac{1}{4}}$ $= 5^{1/7}$

$= \sqrt[7]{5}$

24. $8^{\frac{3}{4}}$

$(2^3)^{3/4} = 2^{9/4} = 2^{2\,1/4}$

$= 2^2\sqrt[4]{2}$

25. $2^{\frac{1}{2}}2^{\frac{3}{4}}$

$2^{2/4}\cdot 2^{3/4} = 2^{5/4} = 2^{1\,1/4}$

$= 2\sqrt[4]{2}$

26. $\sqrt[5]{7}\cdot\sqrt[4]{7^3}$ $7^{1/5}\cdot 7^{3/4}$

$7^{4/20}\cdot 7^{15/20} = 7^{19/20}$

$\sqrt[20]{7^{19}}$

27. $\sqrt[3]{\sqrt{16}}$ $\sqrt[3]{4}$

$\sqrt[3]{4}$

28. $\dfrac{x^{\frac{1}{3}}}{2x^{\frac{2}{3}}}$

$\dfrac{x^{1/3}}{2} = \dfrac{\sqrt[3]{x}}{2}$

29. $\dfrac{14y^{\frac{1}{5}}}{7y^{-\frac{2}{5}}}$

$2y^{3/5} = 2\sqrt[5]{y^3}$

30. $\dfrac{3x^{\frac{2}{3}}}{x^{\frac{1}{6}}}$ $3x^{4/6}$

$3x^{3/6} = 3x^{1/2}$

$= 3\sqrt{x}$

Solve Each Equation.

31. $m^4 - 625 = 0$

$\sqrt[4]{m^4} = \sqrt[4]{625}$

$m = \pm 5, \pm 5i$

32. $\sqrt[3]{(x+4)^3} = \sqrt[3]{-33}$

$x + 4 = \sqrt[3]{-33}$

$x + 4 = -\sqrt[3]{33}$

$x = -4 - \sqrt[3]{33}$ ≈ -7.208

33. $t^4 - 21t^2 + 80 = 0$

$(t^2-16)(t^2-5)=0$

$(t+4)(t-4)(t^2-5)$

$t = -4, 4, \pm\sqrt{5}$

Solve the following equations.

34. $\left(\sqrt{x-3}\right)^2 = (7)^2$

$x - 3 = 49$

$x = 52$

35. $\left(\sqrt[3]{2a+4}\right)^3 = (2)^3$

$2a + 4 = 8$

$2a = 4$

$a = 2$

36. $\sqrt{r+12} - \sqrt{r} = 2$

$\left(\sqrt{r+12}\right)^2 = \left(\sqrt{r}+2\right)^2 \quad (\sqrt{r}+2)$

$r + 12 = r + 2\sqrt{r} + 2\sqrt{r} + 4$

$12 = 4 + 4\sqrt{r}$

$8 = 4\sqrt{r}$

$(2)^2 = (\sqrt{r})^2$

$r = 4$

Solve the following equations.

37. $\sqrt{8x+9}+3=6$

$(\sqrt{8x+9})^2=(3)^2$

$8x+9=9$

$8x=0$

$x=0$

38. $(\sqrt{7x-7})^2=(\sqrt{3x-2})^2$

$7x-7=3x-2$

$4x=5$

$x=5/4$

39. $(x-6)^2=(\sqrt{3x})^2$

$x^2-12x+36=3x$

$x^2-15x+36=0$

$(x-12)(x-3)=0$

$x=12, 3$

$x=12$

40. $2x^{\frac{3}{2}}=16$

$(x^{3/2})^{2/3}=(8)^{2/3}$

$x=(2^3)^{2/3}$

$x=2^2$

$x=4$

41. $(x-5)^{\frac{5}{3}}-73=170$

$((x-5)^{5/3})^{3/5}=(243)^{3/5}$

$x-5=(3^5)^{3/5}$

$x-5=27$

$x=32$

42. $\frac{1}{4}(7x+8)^{\frac{3}{2}}=54$

$((7x+8)^{3/2})^{2/3}=(216)^{2/3}$

$7x+8=(6^3)^{2/3}$

$7x+8=36$

$x=4$

Find the inverse of each function. Then graph the function and its inverse.

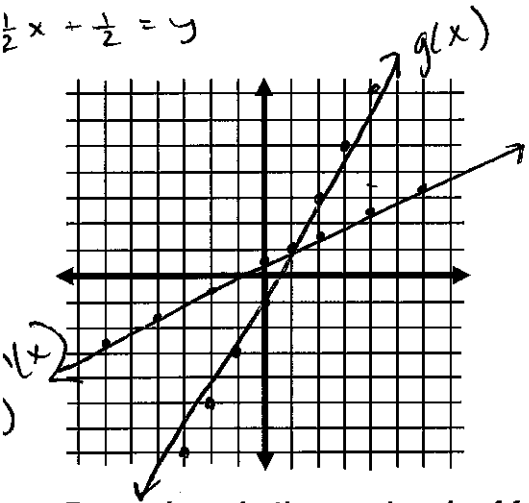
43. $g(x)=2x-1$

$x=2y-1$

$x+1=2y$

$\frac{1}{2}x+\frac{1}{2}=y$

$g^{-1}(x)=\frac{1}{2}x+\frac{1}{2}$

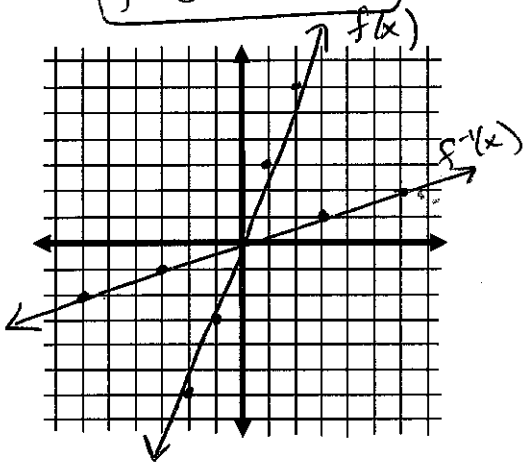


44. $f(x)=3x$

$x=3y$

$y=\frac{1}{3}x$

$f^{-1}(x)=\frac{1}{3}x$



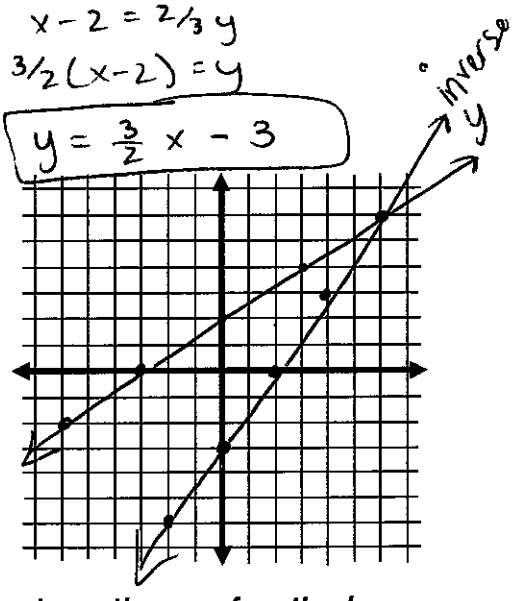
45. $y=\frac{2}{3}x+2$

$x=\frac{2}{3}y+2$

$x-2=\frac{2}{3}y$

$\frac{3}{2}(x-2)=y$

$y=\frac{3}{2}x-3$



Determine whether each pair of functions are inverse functions. You must use the proof method demonstrated in class.

46. $\begin{cases} f(x)=5x-5 \\ g(x)=\frac{1}{5}x+1 \end{cases}$

$f(g(x))=5(\frac{1}{5}x+1)-5$

$=\frac{5}{5}x+5-5$

$=x$ ✓

$g(f(x))=\frac{1}{5}(5x-5)+1$

$=\frac{5}{5}x-1+1$

$=x$ ✓

yes

47. $\begin{cases} f(x)=6x-2 \\ g(x)=\frac{1}{6}x+3 \end{cases}$

$f(g(x))=6(\frac{1}{6}x+3)-2$

$=\frac{6}{6}x+18-2$

$=x+16$ ✗

No

Find the inverse.

48. $f(x)=-\frac{1}{2}x+1$ $x=-\frac{1}{2}y+1$

$2(x-1)=-\frac{1}{2}y \cdot -2$

$-2x+2=y$

$f^{-1}(x)=-2x+2$

49. $g(x)=x^2+7; x\geq 0$ positive $x=y^2+7$

$\sqrt{x-7}=\sqrt{y^2}$

$y=\pm\sqrt{x-7}$

$y=\sqrt{x-7}$

$g^{-1}(x)=\sqrt{x-7}$

Given $f(x)=3x^2-2x+1$ ***and*** $g(x)=x-4$ ***find the following and state the domain.***

50. $f(x)+g(x)$

$(3x^2-2x+1)+(x-4)$

$3x^2-x-3$

$(-\infty,\infty)$

51. $f(x)-g(x)$

$(3x^2-2x+1)-(x-4)$

$3x^2-3x+5$

$(-\infty,\infty)$

52. $f(x)\cdot g(x)$ $(3x^2-2x+1)(x-4)$

$3x^3-12x^2-2x^2+8x+x-4$

$3x^3-14x^2+9x-4$

$(-\infty,\infty)$

53. $\frac{f(x)}{g(x)}$

$\frac{3x^2-2x+1}{x-4}$

$(-\infty,4)\cup(4,\infty)$

54. $f(g(7))$

$g(7)=7-4=3$

$f(3)=3(3)^2-2(3)+1$

$=3(9)-6+1=22$

55. $f(g(x))$

g into f!

$3(x-4)^2-2(x-4)+1$

$3(x^2-8x+16)-2x+8+1$

$3x^2-24x+48-2x+9$

$3x^2-26x+57$ $(-\infty,\infty)$

56. $g(f(x))$ f into g!

$(3x^2-2x+1)-4$

$3x^2-2x-3$ $(-\infty,\infty)$

Given $f(x)=4x^{\frac{1}{3}}+5$ ***and*** $g(x)=-3x^{\frac{1}{3}}-6x$ ***find the following and state the domain.***

57. $f(x)+g(x)$

$4\sqrt[3]{x}+5-3\sqrt[3]{x}-6x$

$\sqrt[3]{x}-6x+5$

$(-\infty,\infty)$

58. $f(x)-g(x)$

$4\sqrt[3]{x}+5+3\sqrt[3]{x}+6x$

$7\sqrt[3]{x}+6x+5$

$(-\infty,\infty)$

59. $g(x)-f(x)$

$-3\sqrt[3]{x}-6x-4\sqrt[3]{x}-5$

$-7\sqrt[3]{x}-6x-5$

$(-\infty,\infty)$

60. $\frac{f(x)}{g(x)}$

$\frac{4\sqrt[3]{x}+5}{-3\sqrt[3]{x}-6x}$

$(-\infty,0)\cup(0,\infty)$

61. $\frac{g(x)}{f(x)}$

$\frac{-3\sqrt[3]{x}-6x}{4\sqrt[3]{x}+5}$

$4\sqrt[3]{x}\neq -5$

$\sqrt[3]{x}\neq -5/4$

$x\neq -125/64$

$(-\infty,-\frac{125}{64})\cup(-\frac{125}{64},\infty)$

Algebra 2 CP

Midterm Review Chapter 7

Name Key

Determine whether each function represents exponential growth or decay.

1. $y = 5(.6)^x$ Decay

2. $y = .1(2)^x$ Growth

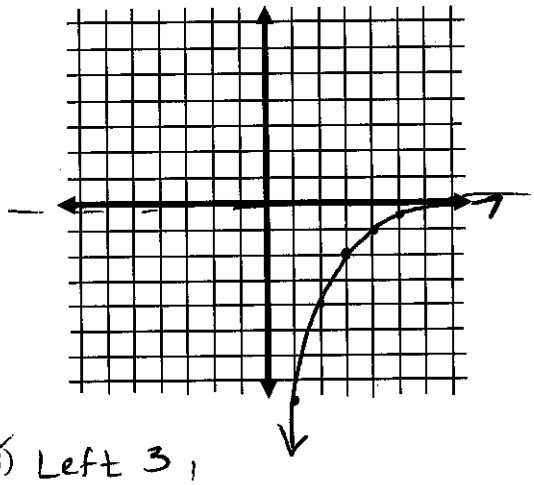
3. $y = 5 \cdot 4^{-x}$ Decay

Describe the shifts of the following exponential equations from its parent function and then graph. Then, identify the functions domain and range.

4. $y = -2\left(\frac{1}{2}\right)^{x-3}$

Reflect over x
Right 3

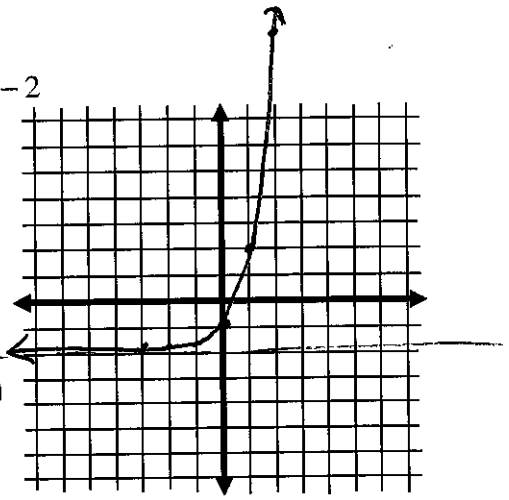
D: $(-\infty, \infty)$
R: $(-\infty, 0)$



5. $y = .25(4)^{x+1} - 2$

Left 1
Down 2

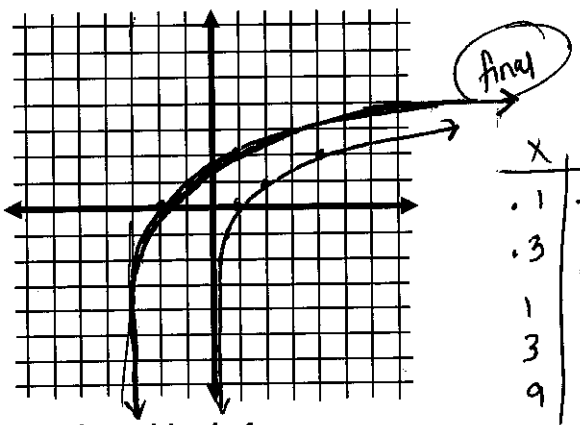
D: $(-\infty, \infty)$
R: $(-2, \infty)$



6. $y = \log_2(x + 3)$ Left 3

$2^y = x$

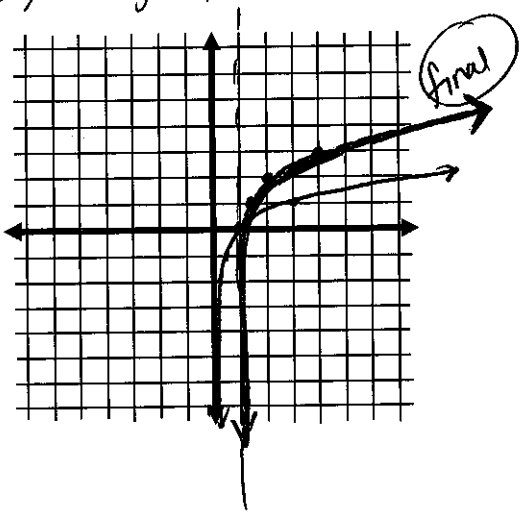
x	y
.25	-2
.5	-1
1	0
2	1
4	2



7. $y = \log_3(x - 1) + 2$ Right 1, Up 2

$3^y = x$

x	y
.1	-2
.3	-1
1	0
3	1
9	2



Write each expression in logarithmic form.

8. $2^7 = 128$ $\log_2 128 = 7$

9. $8^{-2} = \frac{1}{64}$ $\log_8 \frac{1}{64} = -2$

Write each equation in exponential form.

10. $\log_{15} 225 = 2$ $15^2 = 225$

11. $\log_4 32 = \frac{5}{2}$ $4^{5/2} = 32$

Write an equivalent exponential or logarithmic equation.

12. $e^{15} = x$ $\ln x = 15$

13. $\ln 20 = x$ $e^x = 20$

14. $e^{-5x} = 0.2$ $\ln 0.2 = -5x$

15. $\ln 0.0002 = x$ $e^x = 0.0002$

Evaluate each expression.

16. $\log_5 25 = x$ $5^x = 25$ $x = 2$

17. $\log_4 \frac{1}{64} = x$ $4^x = 1/64$ $x = -3$

18. $7^{\log_7 x}$ x

19. $e^{\ln 3}$ 3

20. $\ln e^y$ y

Solve each equation. Check your solutions.

21. $\log_3 x = 5$ $3^5 = x$ $x = 243$

22. $\log_b 3 = \frac{1}{2}$ $(b^{1/2})^2 = (3)^2$ $b = 9$

23. $\log_6 216 = x$ $6^x = 216$ $x = 3$

24-27, use $\log_5 3 \approx 0.6826$ ***and*** $\log_5 4 \approx 0.8614$ ***to evaluate each expression.***

24. $\log_5 12$ $\log_5 3 \cdot 4 = \log_5 3 + \log_5 4$ $.6826 + .8614 = 1.544$

25. $\log_5 \frac{81}{5}$ $= \log_5 \frac{3^4}{5} = 4\log_5 3 - \log_5 5$ $= 4(.6826) - 1$ $= 1.7304$

26. $\log_5 \frac{9}{16}$ $= \log_5 \frac{3^2}{4^2} = 2\log_5 3 - 2\log_5 4$ $= 2(.6826) - 2(.8614)$ $= -0.3576$

27. $\log_5 144$ $\log_5 3^2 \cdot 4^2 = 2\log_5 3 + 2\log_5 4$ $= 2(.6826) + 2(.8614)$ $= 3.088$

Use the Change of Base formula to evaluate.

28. $\log_5 7$ $\frac{\log 7}{\log 5} = 1.209$

29. $\log_9 4$ $\frac{\log 4}{\log 9} = 0.631$

Expand the following logarithms.

30. $\log_3 4x^2$ $\log_3 4 + 2\log_3 x$

31. $\ln \frac{3x^3}{2y}$ $\ln 3 + 3\ln x - \ln 2 - \ln y$

Condense the following logarithms.

32. $\log_5 24 - \log_5 6$ $\log_5 \frac{24}{6} = \log_5 4$

33. $\log_8 6 + 2\log_8 3$ $\log_8 6 + \log_8 3^2$ $\log_8 54$

Solve each equation or inequality. Check your solution.

34. $3^{3x-5} = 81$ $3^{3x-5} = 3^4$ $3x - 5 = 4$ $3x = 9$ $x = 3$

35. $2^{3n-1} = \left(\frac{1}{8}\right)^n$ $2^{3n-1} = (2^{-3})^n$ $3n - 1 = -3n$ $6n = 1$ $n = 1/6$

36. $9^{2x-1} = 27^{x+4}$ $(3^2)^{2x-1} = (3^3)^{x+4}$ $4x - 2 = 3x + 12$ $x = 14$

37. $5^x + 3 = 12$

$5^x = 9$

$\log 5^x = \log 9$

$x \cdot \frac{\log 5}{\log 5} = \frac{\log 9}{\log 5}$

$x = 1.365$

38. $7^{2x-1} + 5 = 27$

$7^{2x-1} = 22$

$(2x-1)\frac{\log 7}{\log 7} = \frac{\log 22}{\log 7}$

$2x - 1 = 1.588$

$x = 1.294$

39. $e^x = 5$

$\ln e^x = \ln 5$

$x \cdot \ln e = \ln 5$

$x = 1.609$

40. $2e^x - 1 = 11$

$2e^x = 12$

$e^x = 6$

$\ln e^x = \ln 6$

$x \cdot \ln e = \ln 6$

$x = 1.792$

Solve each equation. Check your solutions.

41. $\log_6(4x+12) = 2$

$6^2 = 4x + 12$

$36 = 4x + 12$

$24 = 4x$

$x = 6$

42. $\log_3(x+2) = \log_3(3x)$

$x + 2 = 3x$

$2 = 2x$

$x = 1$

43. $\log_4 5 + \log_4 x = \log_4 60$

$\log_4 5 \cdot x = \log_4 60$

$5x = 60$

$x = 12$

44. $\log_5 y - \log_5 8 = \log_5 1$

$\log_8 \frac{y}{8} = \log_8 1$

$\frac{y}{8} = 1$

$y = 8$

45. $3\log_8 2 - \log_8 4 = \log_8 b$

$\log_8 2^3 - \log_8 4 = \log_8 b$

$\log_8 \frac{8}{4} = \log_8 b$

$2 = b$

46. $\log_3 x - \log_3(x+2) = 1$

$\log_3 \frac{x}{x+2} = 1$

$(x+2)3^1 = \frac{x}{x+2}(x+2)$

$3x + 6 = x$

$6 = -2x$

$x = -3$

No Solution

47. $\log_2(x-2) + \log_2(x+1) = 2$

$\log_2(x^2 - x - 2) = 2$

$2^2 = x^2 - x - 2$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3, -2$

$x = 3$

48. $\ln e^{8x} = 3$

$e^3 = 8x$

$20.0855 = 8x$

$x = 2.511$

49. $\ln_e(x+3) = 1$

$e^1 = x + 3$

$2.718 = x + 3$

$x = -.282$

Find the inverse.

50. $y = 4^x$

$x = 4^y$

$y = \log_4 x$

51. $y = \ln(x+3)$

$x = \ln_e(y+3)$

$e^x - 3 = y$

52. $y = \log_8 x$

$x = \log_8 y$

$8^x = y$

53. A computer system depreciates at a rate of 6.5% per year. If the computer system originally cost $4000, how long would it take for it to be worth half its value?

$a = 4000$

$y = \frac{1}{2}(4000) = 2000$

$r = .065$

$t = ?$

$2000 = 4000(1-.065)^t$

$.5 = (.935)^t$

$\log .5 = \log .935^t$

$\log .5 = t \cdot \log .935$

$t = 10.3$ years

54. Suppose you deposit $1000 in an account paying 5% annual interest compounded continuously. $y = Pe^{rt}$

a. What is the balance after 10 years?

$y = 1000e^{.05(10)}$

$y = 1648.72$

$\$1648.72$

b. How long will it take for the balance in your account to reach $1500?

$\frac{1500}{1000} = \frac{1000e^{.05t}}{1000}$

$1.5 = e^{.05t}$

$\ln 1.5 = \ln e^{.05t}$

$\ln 1.5 = .05t \ln e$

$.405 = .05t$

$t = 8.1$ years

55. Suppose you deposit $100 in an account paying 3.5% interest compounded continuously. How long will it take for your money to double? $y = 200$ $y = Pe^{rt}$

$200 = 100e^{.035t}$

$2 = e^{.035t}$

$\ln 2 = \ln e^{.035t}$

$\ln 2 = .035t \ln e$

$.693 = .035t$

$t = 19.8$ years

56. A cup of coffee contains ~~130~~ milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?

$65 = 130(1-.11)^t$

$.5 = (.89)^t$

$\log .5 = \log .89^t$

$\frac{\log .5}{\log .89} = \frac{t \cdot \log .89}{\log .89}$

$t = 5.9$ hours

57. You're off to college! You buy a computer for $2500. It is expected to depreciate at a rate of 20% per year. What will be the value of the computer in 2 years?

$y = 2500(1-.2)^2$

$= 2500(.8)^2$

$y = \$1600$

58. A computer system depreciates at an average rate of 4% per month. If the value of the computer system was originally $12000, in how many months is it worth $7350?

$\frac{7350}{12000} = \frac{12000(1-.04)^t}{12000}$

$.6125 = (.96)^t$

$\log .6125 = t \times \log .96$

$t = 12.0$ months

59. A piece of machinery valued at $250,000 depreciates at a fixed rate of 12% per year. After how many years will the value have depreciated to $100,000?

$\frac{100,000}{250000} = \frac{250,000(1-.12)^t}{250000}$

$.4 = (.88)^t$

$\log .4 = t \times \log .88$

$t = 7.2$ years

60. The Miller's bought a condominium for $185,000. Assuming that the value of the condo will appreciate at approximately 5% a year, how much will the condo be worth in 7 years?

$y = 185,000(1+.05)^7$

$= 185000(1.05)^7$

$y = \$260,313.58$

61. The population of a city of one million people is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled?

$2 = 1(1.03)^t$

$\log 2 = t \times \log 1.03$

$t = 23.4$ years

Algebra 2 CP

Midterm Review Chapter 11

Name Key

1. Of the 42 employees at Speedy Pizza, sixteen make $4.75 an hour, four earn $5.50 and hour, three earn $6.85 an hour, six earn $4.85 an hour and thirteen earn $5.25 an hour. Find the mean, median and mode of the hourly wages.

$16(4.75) + 4(5.5) + 3(6.85) + 6(4.85) + 13(5.25) = \frac{215.9}{42} = \$5.14$

Mean: $5.14

Median: $4.85

Mode: $4.75

4.75	4.85	5.25	5.5	6.85
16	6	13	4	3

2. Given the following set of data:

32, 45, 67, 93, 82, 56, 58, 45, 13, 54

a) Make a stem and leaf plot of this data.

Stem	Leaf
1	3
2	
3	2
4	5 5
5	4 5 8
6	7
7	
8	2
9	3

b) What is the minimum? b) 13

c) What is the maximum? c) 93

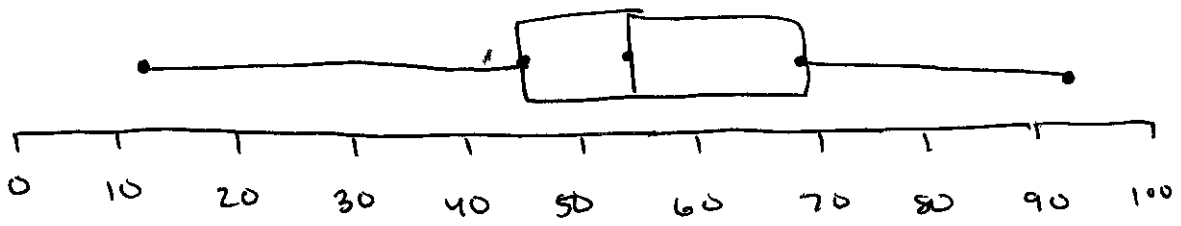
d) What is the range? $93 - 13 = 80$ d) 80

e) What is the mean? (to nearest tenth) $544/10 = 54.4$ e) 54.4

f) What is the mode? f) 45

g) What is the median? g) 54.5

h) Construct a box plot of the data.



j) What is the lower quartile? j) 45

k) What is the upper quartile? k) 67

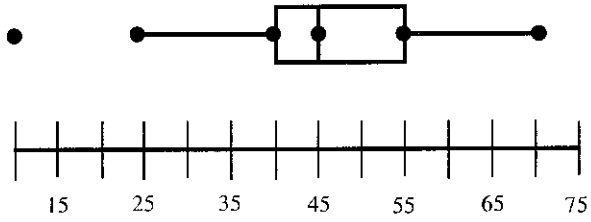
l) What is the inter-quartile range? (IQR) l) 22

m) What is the outlier number? $1.5 \times 22 = 33$ m) 33

n) What is the lower fence? $45 - 33$ n) 12

o) What is the upper fence? $67 + 33$ o) 100

p) Name the outlier(s) if any. p) None



3. From the box plot, answer the following questions.

a) What is the lower quartile? a) 40

b) What is the upper quartile? b 55

c) What is the inter-quartile range? (IQR) c) 15

d) What is the outlier number? $15 \times 1.5 = 22.5$ d) 22.5

e) What is the lower fence? $40 - 22.5$ e) 17.5

f) What is the upper fence? $55 + 22.5$ f) 57.5

g) Name the outlier(s) if any. g) 10

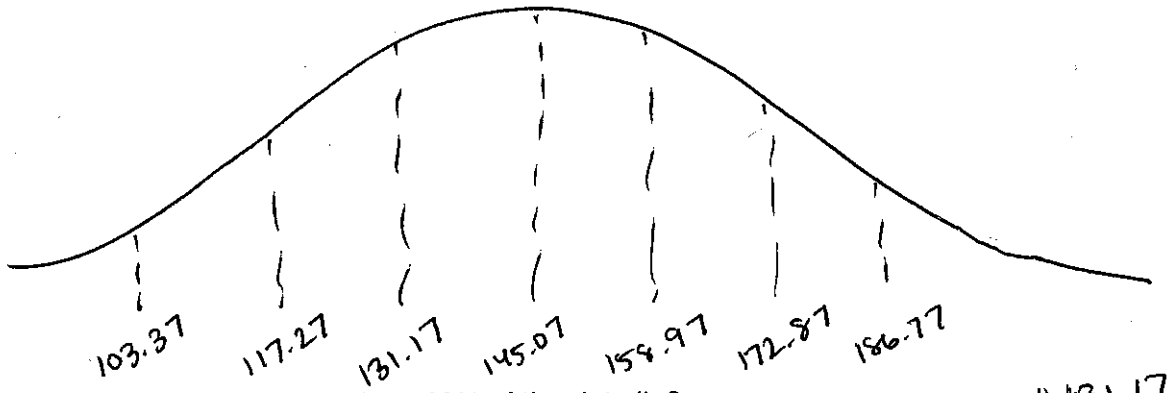
4) Below are the weights of 14 people boarding a sail boat.

130, 160, 135, 139, 121, 135, 142, 156, 150, 169, 162, 134, 143, 155

a) What is the mean weight? $2031/14$ a) 145.07

b) What is the standard deviation of the weights? calc b) 13.9

c) Construct a normal distribution of the weights.



d) Between what 2 numbers does 68% of the data lie? d) 131.17, 158.97

e) Between what 2 numbers does 95% of the data lie? e) 117.27, 172.87

f) Above what number can we find weights that are 3 or more standard deviations above of the mean? f) 186.77

g) Calculate the z-score for a person weighing 135 pounds. $\frac{135-145.07}{13.9}$ g) −0.72

h) What is the probability for a person to have a z-score of -2.5? h) .0062 → .62%.

Use Table!!!

5) 323 people were surveyed. 46% responded that they did use e-mail.

a) What is the margin of error for this survey? $\pm \frac{1}{\sqrt{n}} = \pm \frac{1}{\sqrt{323}}$ a) ± 5.56%

b) Give an interval that is likely to contain the exact percentage of people. b) 40.44% − 51.56%

c) If we wanted a margin of error of ± 5%, what would the sample size need to be? c) 400 people

$$.05 = \pm \frac{1}{\sqrt{n}}$$

$$.05\sqrt{n} = \pm 1$$

$$(\sqrt{n})^2 = \left(\pm \frac{1}{.05}\right)^2$$

$n = 400$ people