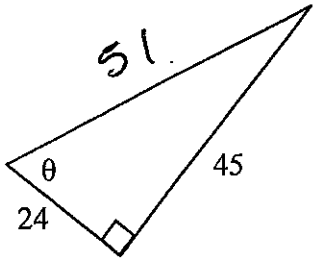
Algebra 2 CP Final Exam Review Name _____
2008-2009

Chapter 13 TRIGONOMETRY

Note: always round sides to the nearest tenth and angles to the nearest degree. * ordered pair

1-2 Find the values of the six trigonometric functions for angle θ.

1)



1) $\sin\theta$ $45/51 = 15/17$

$\cos\theta$ $24/51 = 8/17$

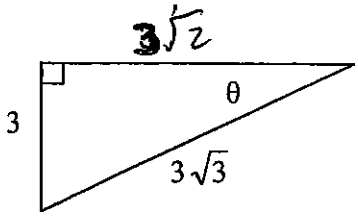
$\tan\theta$ $45/24 = 15/8$

$\csc\theta$ $51/45 = 17/15$

$\sec\theta$ $51/24 = 8/17$

$\cot\theta$ $24/45 = 15/8$

2)



$3^2 + b^2 = (3\sqrt{3})^2$

$b^2 = 18$

$b = 3\sqrt{2}$

$\sin\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

$\tan\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

2) $\sin\theta$ $\sqrt{3}/3$

$\cos\theta$ $\sqrt{6}/3$

$\tan\theta$ $\sqrt{2}/2$

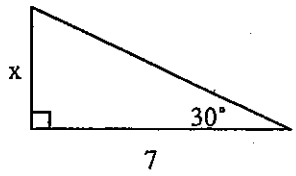
$\csc\theta$ $\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\sec\theta$ $\frac{3}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$

$\cot\theta$ $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

3-4, Write an equation involving sin, cos, or tan that can be used to find x, then solve.

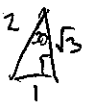
3)



$\tan 30 = \frac{x}{7}$

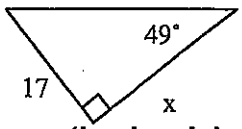
$x = 7\tan 30$

$= 7\left(\frac{\sqrt{3}}{3}\right) = \frac{7\sqrt{3}}{3}$



3) $\frac{7\sqrt{3}}{3}$

4)

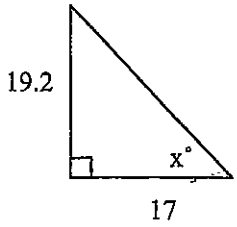


$\tan 49 = \frac{17}{x}$ $x = \frac{17}{\tan 49}$

4) 14.8

Write an equation involving sin, cos, or tan that can be used to find x, then solve.

5)



$\tan x = \frac{19.2}{17}$

$x = 48°$

5) 48°

6-7, Solve ΔABC by using the given measurements.

6) $A = 35^\circ$, $a = 12$

$\tan 35 = \frac{12}{b}$ $\sin 35 = \frac{12}{c}$

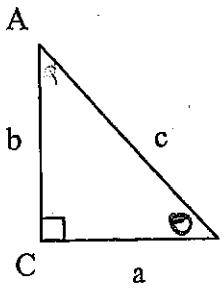
$b = 17.1$ $c = 20.9$

7) $b = 52$, $c = 95$

$a = 79.5$

$\sin A = \frac{79.5}{95}$

$A = 57^\circ$

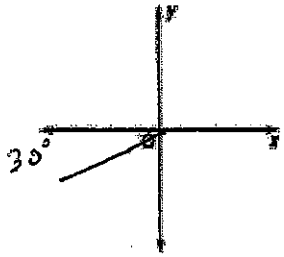


6) $B = 55$
$b = 17.1$
$c = 20.9$

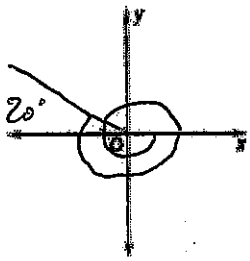
7) $a = 79.5$
$A = 57^\circ$
$B = 33^\circ$

8-9, Draw an angle with the given measure in standard position.

8) 210°



9) -560°



10-13, Rewrite each degree measure in radians and each radian measure in degrees.

10) $-18^\circ \cdot \frac{\pi}{180} = -\frac{\pi}{10}$

11) $870^\circ \cdot \frac{\pi}{180} = \frac{29\pi}{6}$

12) $\frac{5\pi}{2} \cdot \frac{180}{\pi} = 450$

13) $-\frac{7\pi}{12} \cdot \frac{180}{\pi}$

10) $-\pi/10$

11) $\frac{29\pi}{6}$

12) 450°

13) -105°

14-17, Find one angle with positive measure and one angle with negative measure coterminal with each angle.

14) 80°

15) $\frac{2\pi}{5}$

16) -93°

17) $-\frac{5\pi}{12}$

14) $440, -280$

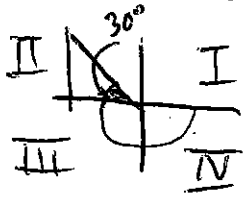
15) $\frac{12\pi}{5}, -\frac{8\pi}{5}$

16) $267^\circ, -453^\circ$

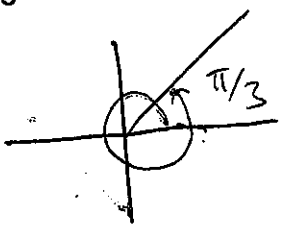
17) $\frac{19\pi}{12}, -\frac{29\pi}{12}$

18-19, Find the reference angle for the angle with the given measure.

18) -210°



19) $\frac{13\pi}{3}$



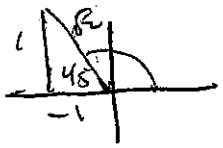
18) 30°

19) $\pi/3$

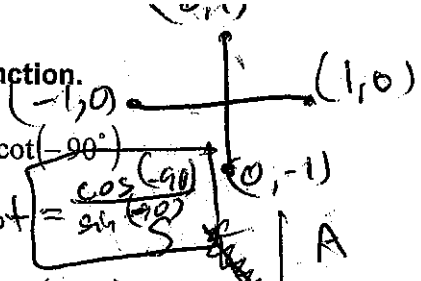
Find the exact value of each trigonometric function.

$\tan = \frac{\sin}{\cos}$

20) $\tan 135^\circ$

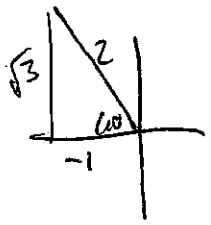


21) $\cot(-90^\circ)$

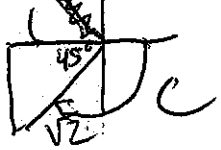


$\cot = \frac{\cos(-90)}{\sin(-90)} = \frac{0}{-1}$

22) $\tan\frac{5\pi}{3}$



23) $\csc\left(-\frac{3\pi}{4}\right)$



$\frac{1}{\sin 45} = \frac{1}{\frac{1}{\sqrt{2}}} = -\sqrt{2}$

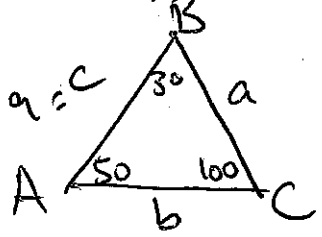
20) -1

21) 0

22) $-\sqrt{3}$

23) $-\sqrt{2}$

24) Solve ΔABC if $A = 50^\circ$, $B = 30^\circ$ and $c = 9$.



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 50} = \frac{9}{\sin 100}$

$a \sin 100 = 9 \sin 50$ $a = 7$

$\frac{b}{\sin 30} = \frac{9}{\sin 100}$ $b = 4.6$

24) C = 100°

a = 7

b = 4.6

25-27, Determine whether each triangle has one, two or no solutions, then solve each triangle.

25) $A = 29^\circ$, $a = 6$ and $b = 13$

$\frac{6}{\sin 29} = \frac{13}{\sin B}$ $\sin B = 1.05$

$B = \sin^{-1}(1.05)$

25) No sol

26) $A = 66^\circ$, $a = 12$ and $b = 7$

$\frac{12}{\sin 66} = \frac{7}{\sin B}$ $\sin B = \frac{7 \sin 66}{12}$

$B = \sin^{-1}(.53)$

$B = 32^\circ$

$\frac{7}{\sin 32} = \frac{c}{\sin 82}$

$c = 13$

26) $B = 32^\circ$

$C = 82^\circ$ $c = 13$

~~$A = 66$~~ ~~$B = 48$~~ ~~$c = 34$~~

27) $A = 45^\circ$, $a = 15$ and $b = 18$ $\frac{15}{\sin 45} = \frac{18}{\sin B}$; $B \approx 58^\circ$

Case 1

$A = 45$ $a = 15$

$B = 58$ $b = 18$

$C = 77$ $c = 20.7$

$\frac{15}{\sin 45} = \frac{c}{\sin 77}$ $c = 20.7$

Case 2

$A = 45$ $a = 15$

$B = 122$ $b = 18$

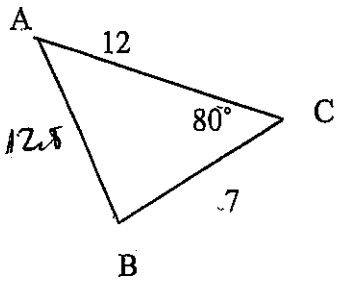
$C = 13$ $c = 4.8$

$\frac{15}{\sin 45} = \frac{c}{\sin 13}$

27)

28-29, Solve the following triangles.

28)



$c^2 = 12^2 + 7^2 - 2(12)(7)\cos 80$

$c^2 = 163.8$

$c = 12.8$

$\frac{12.8}{\sin 80} = \frac{7}{\sin A}$

$\sin A = .54$

$A = 33°$

28) $c = 12.8$

$A = 33°,\ B = 67°$

29) $a = 16,\ b = 20,\ C = 54°$.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 16^2 + 20^2 - 2(16)(20)\cos 54$

$c^2 = 420.8$

$c = 20.5$

$\frac{20.5}{\sin 54} = \frac{16}{\sin A}$

$\sin A = .63$

$A = 39°$

29) $c = 20.5$

$A = 39°,\ B = 87°$

30-31, Solve the following triangles.

30) $a = 8,\ b = 6,\ c = 9$

$9^2 = 8^2 + 6^2 - 2(8)(6)\cos C$

$81 = 100 - 96\cos C$

$-.19 = \cos C$

$C = 79°$

$\frac{8}{\sin A} = \frac{9}{\sin 79}$

$\sin A = .87$

$A = 61°$

30) $C = 79°,\ A = 61°$

$B = 40°$

31) $B = 47°,\ C = 112°,\ b = 13$

$\frac{13}{\sin 47} = \frac{c}{\sin 112}$

$c = 16.5$

$\frac{a}{\sin 21} = \frac{13}{\sin 47}$

$a = 6.4$

31) $c = 16.5,\ A = 21°$

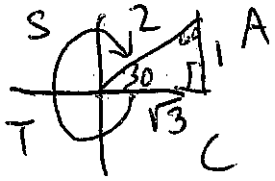
$a = 6.4$

32-35, find the exact value of each function.

32) $\cos\frac{7\pi}{4}$

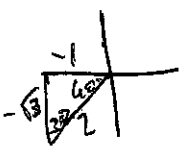
$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

33) $\sin(-330°)$



$= \frac{1}{2}$

34) $\sin\left(-\frac{2\pi}{3}\right)$



$\frac{-\sqrt{3}}{2}$

35) $\cos 840°$

$= -\frac{1}{2}$

32) $\frac{\sqrt{2}}{2}$

33) $\frac{1}{2}$

34) $\frac{-\sqrt{3}}{2}$

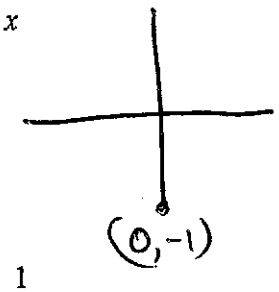
35) $-\frac{1}{2}$

sin + tan defined $-90 \to 90$ cos $0 \to 180$

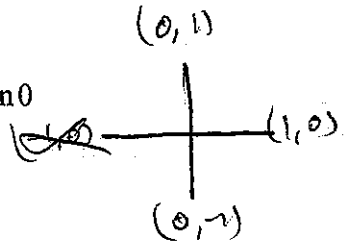
36-39, Solve each equation by finding the value of x.

36) $Sin^{-1}(-1) = x$

$-90°$



37) $x = Arc\tan 0$

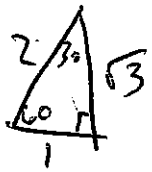


36) $-90°, -\pi/2$

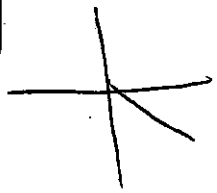
37) $0°, 0$

38) $x = Arc\cos\frac{1}{2}$

$60°$



39) $Arc\tan\left(-\frac{\sqrt{3}}{3}\right)$

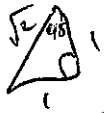


38) $60°, \pi/3$

39) $-30°, -\pi/6$

Find each value. Write angle measures in radians.

40) $Sin^{-1}\frac{\sqrt{2}}{2}$



41) $Tan^{-1}(-\sqrt{3})$

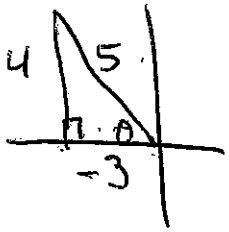
40) $45°, \pi/4$

41) $-60°, -\pi/3$

Find the value of each expression.

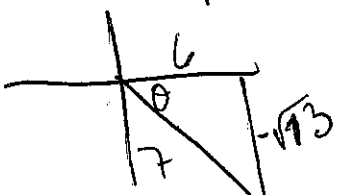
42) $\cos\theta$, if $\tan\theta = -\frac{4}{3}$; $90° < \theta < 180°$

$\cos\theta = \frac{-3}{5}$



43) $\sin\theta$, if $\cos\theta = \frac{6}{7}$; $270° < \theta < 360°$

$\sin\theta$

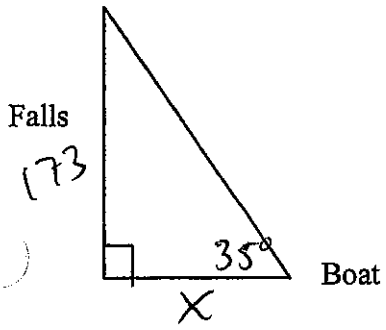


42) $-3/5$

43) $-\frac{\sqrt{13}}{7}$

44) In a sightseeing boat near the base of the Horseshoe Falls at Niagara Falls, a passenger estimates the angle of elevation to the top of the Falls to be 35°. If the Horseshoe Falls are 173 feet high, what is the distance from the boat to the base of the falls?

44) 247 ft



$\tan 35 = \frac{173}{x}$

$x = \frac{173}{\tan 35}$ $x = 247$ ft

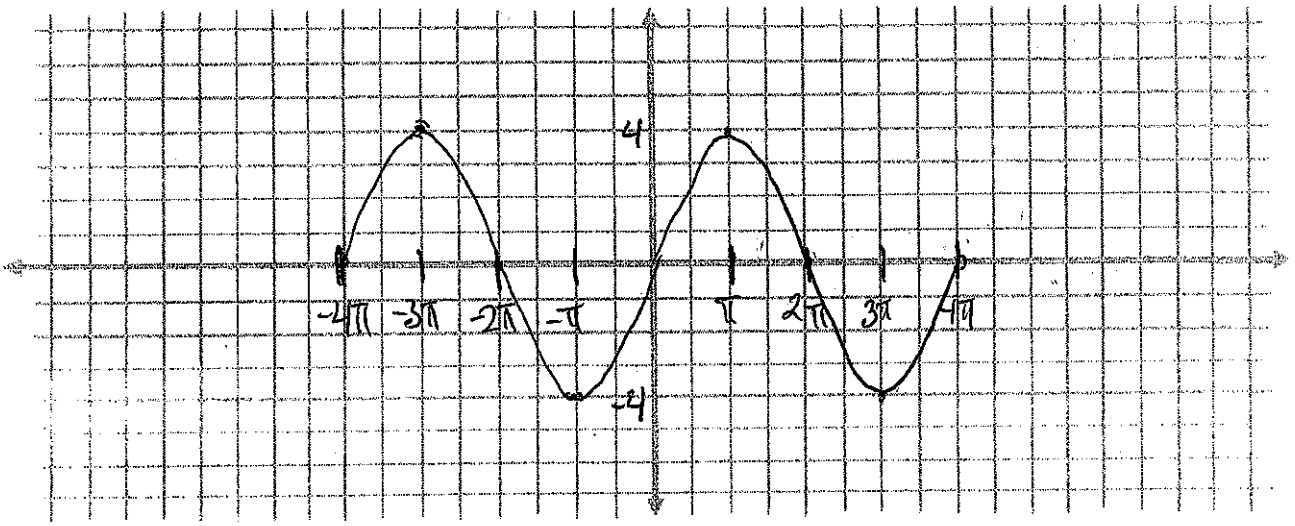
Chapter 14 GRAPHING SINE AND COSINE

45) Given, $y = 4\sin\frac{1}{2}\theta$ find the following in radians:

a) Amplitude 4 — 45 a) _____

b) Period $\frac{2\pi}{1/2} = 4\pi$ — b) _____

c) Graph one positive and one negative period. (be sure to label graph)

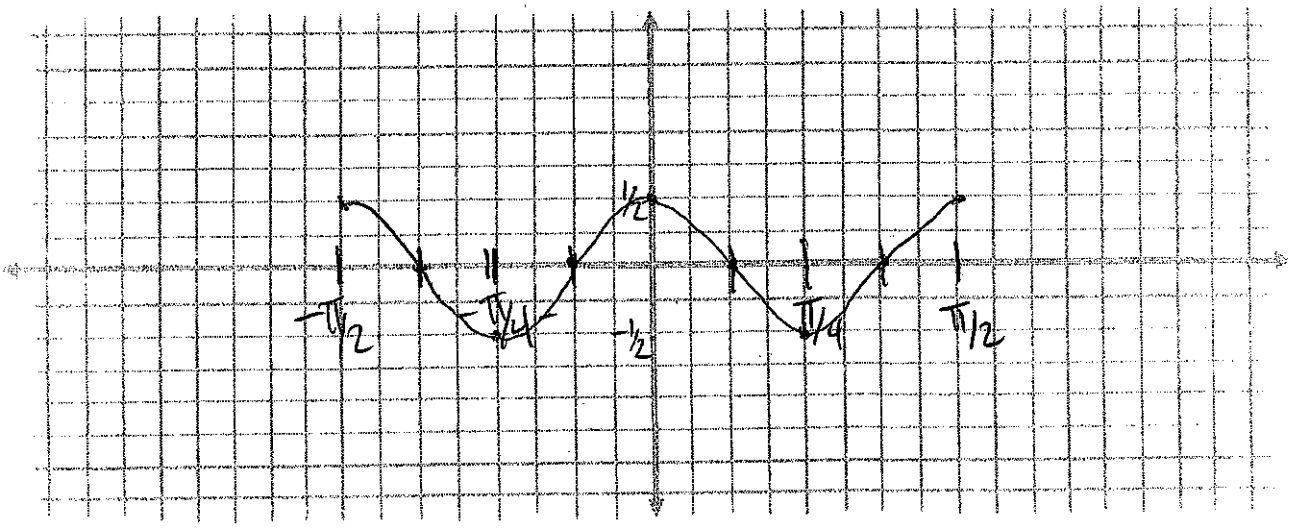


46) Given, $y = \frac{1}{2}\cos 4\theta$ find the following in radians:

a) Amplitude 1/2 — 46 a) _____

b) Period $\frac{2\pi}{4} = \frac{\pi}{2}$ — b) _____

c) Graph one positive and one negative period. (be sure to label graph)



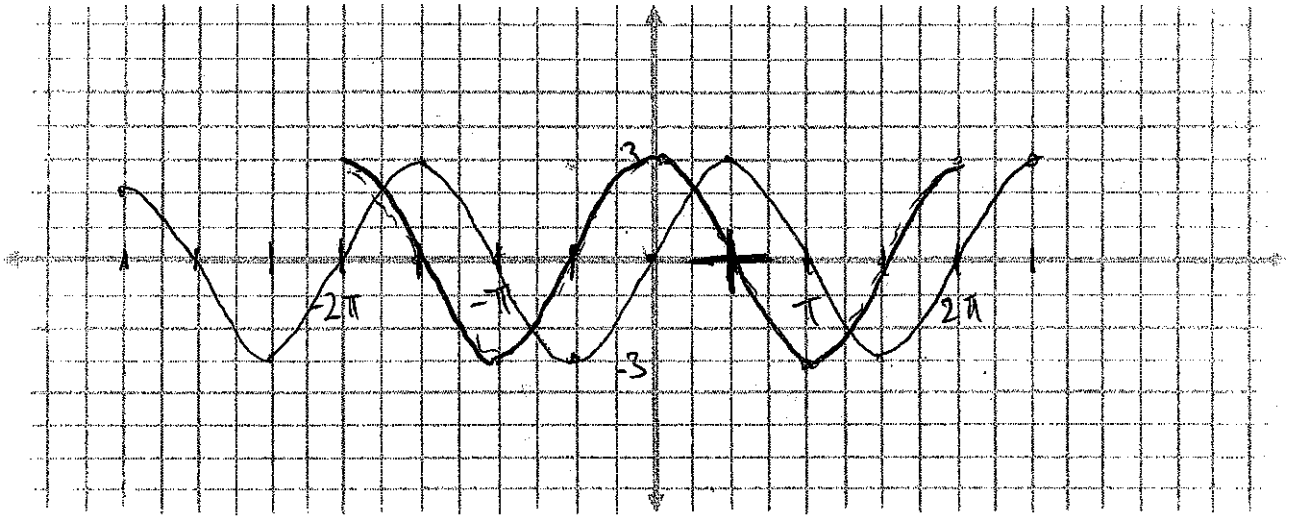
47) Given, $y = 3\cos\left(\theta - \frac{\pi}{2}\right)$ find the following:

a) Amplitude 3 — 47 a) _____

b) Period 2π — b) _____

c) Phase Shift right $\pi/2$ — c) _____

d) Graph one positive and one negative period. (be sure to label graph)



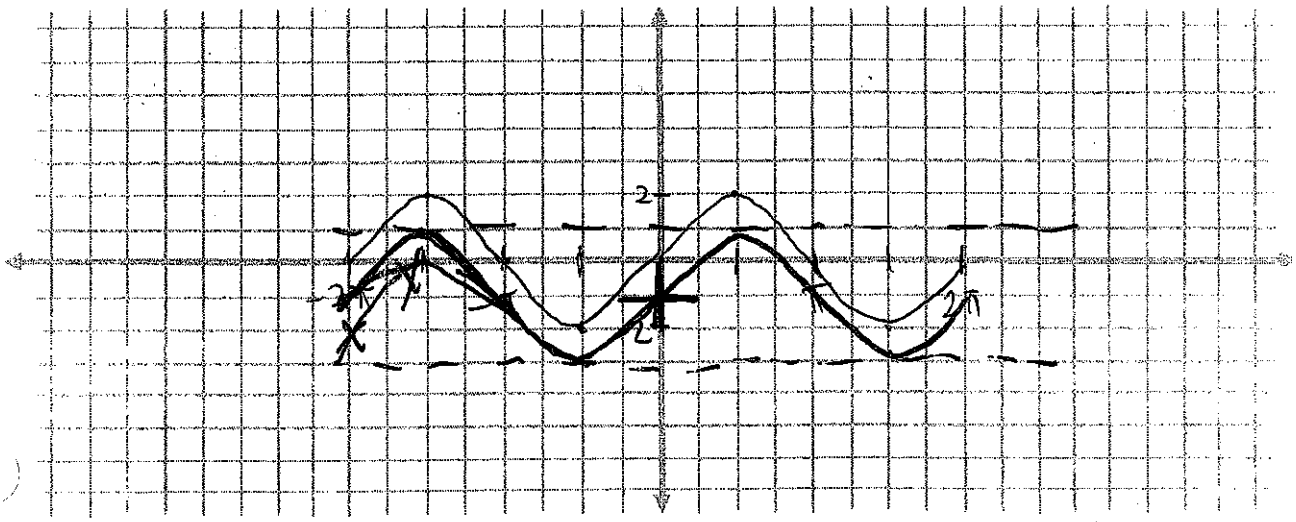
) Given, $y = 2\sin\theta - 1$ find the following in radians:

a) Amplitude 2 — 48 a) _____

b) Period 2π — b) _____

c) Vertical Shift down 1 — c) _____

d) Graph one positive and one negative period. (be sure to label graph)



Chapter 8 RATIONAL EXPRESSIONS

Simplify each expression.

49) $\frac{21x^3y}{14x^2y^2} = \frac{3x}{2y}$

50) $\frac{x^2+x-6}{x^2-6x-27} = \frac{(x+3)(x-2)}{(x-9)(x+3)}$

49) $\frac{3x}{2y}$

50) $\frac{x-2}{x-9}$

51) $\frac{(m-3)^2}{m^2-6m+9} \cdot \frac{m^3-9m}{m^2-9}$

$= \frac{(m-3)(m-3) \cdot m(m^2-9)}{(m-3)(m-3)(m^2-9)}$

$= m$

52) $\frac{c^2-3c}{c^2-25} \cdot \frac{c^2+4c-5}{c^2-4c+3} = \frac{c(c-5)(c+5)(c-1)}{(c-5)(c+5)(c-3)(c-1)}$

$= \frac{c}{c-5}$

51) m

52) $\frac{c}{c-5}$

53) $\frac{6xy^4}{25z^3} \div \frac{18xz^2}{5y} = \frac{6xy^4}{5 \cdot 25z^3} \cdot \frac{5y}{3 \cdot 18xz^2}$

$= \frac{y^5}{15z^5}$

54) $\frac{16p^2-8p+1}{14p^4} \div \frac{4p^2+7p-2}{7p^5}$

$\frac{(4p-1)(4p-1)}{2 \cdot 7p^4} \cdot \frac{7p^5}{(4p-1)(p+2)}$

$= \frac{p(4p-1)}{2(p+2)}$

53) $\frac{y^5}{15z^5}$

54) $\frac{p(4p-1)}{2(p+2)}$

55) $\frac{3}{8p^2q} + \frac{5}{4p^2q} \cdot \frac{2}{2} = \frac{3+10}{8p^2q}$

$= \frac{13}{8p^2q}$

56) $\frac{4z}{z-4} + \frac{z+4}{z+1}$ $\quad (z+1)(z-4)$

$= \frac{4z(z+1)+(z+4)(z-4)}{(z-4)(z+1)}$

$\frac{4z^2+4z+z^2-16}{(z-4)(z+1)} = \frac{5z^2+4z-16}{(z-4)(z+1)}$

55) $\frac{13}{8p^2q}$

56) $\frac{5z^2+4z-16}{(z-4)(z+1)}$

57) $\frac{3}{(w+3)} - \frac{2}{w^2-9}$ $\quad (w+3)(w-3)$

$= \frac{3w+9-2}{w^2-9}$

$= \frac{3w+7}{w^2-9}$

58) $\frac{5}{3b+d} - \frac{2}{3bd}$ $\quad 3bd(3b+d)$

$= \frac{15bd-6b+2d}{3bd(3b+d)}$

57) $\frac{3w+7}{(w-3)(w+3)}$

58) $\frac{15bd-6b+2d}{3bd(3b+d)}$

Determine any value(s) of x that are undefined. ← vertical asymptotes

59) $f(x) = \frac{3x-1}{3x^2+5x-2}$

$(3x-1)(x+2) = 0$

$x = \frac{1}{3}, -2$

60) $f(x) = \frac{x^2-x-12}{x^2-4x}$

$x(x-4) = 0$

$x = 0, 4$

59) $\frac{1}{3}, -2$

60) $0, 4$

Solve the following.

61) $\frac{3}{x+1} = \frac{9}{4x+5}$

$9x+9 = 12x+15$

$-3x = 6$

$x = -2$

61) $x = -2$

62) $\frac{3}{x} + \frac{4}{x-1} = \frac{x+1}{x-1}$

$3(x-1) + 8 = 2x+2$

$3x-3+8 = 2x+2$

$3x+5 = 2x+2$

$x = -3$

62) $x = -3$

63) $1 - \frac{8}{x-5} = \frac{3}{x}$

$x^2 - 5x - 8x = 3x - 15$

$x^2 - 13x = 3x - 15$

$x^2 - 16x + 15 = 0$

$(x-1)(x-15) = 0$

$x = 1, 15$

63) $x = 1, 15$

64) $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$

$6x+18 = 8x^2 - 4x^2 + 12x$

$0 = 4x^2 + 6x - 18$

$0 = 2(2x^2 + 3x - 9)$

$0 = 2(2x-3)(x+3)$

$x = 3/2, -3$

64) $x = 3/2$

65) $\frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$

$(x^2+x) + (x+6) = 2x^2 + x$

$x^2 + 2x + 6 = 2x^2 + x$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

66) $\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$ → $2x + x - 3 = x^2 - x$

$3x - 3 = x^2 - x$

$0 = x^2 - 4x + 3$

$0 = (x-3)(x-1)$

65) $x = 3, -2$

66) $x = 1$

67) Find the product: $\frac{x^2-11x+24}{x^2-18x+80} \cdot \frac{x^2-15x+50}{x^2-9x+20}$

$\frac{(x-3)(x-8)}{(x-8)(x-10)} \cdot \frac{(x-5)(x-10)}{(x-5)(x-4)} = \frac{x-3}{x-4}$

67.) $\frac{x-3}{x-4}$

68) Solve: $\frac{2}{x-1} = 4 - \frac{x}{x-1}$

$2 = 4x - 4 - x$

$2 = 3x - 4$

$6 = 3x$

$x = 2$

68.) $x = 2$

69) Solve: $\frac{9}{x-3} = \frac{x-4}{x-3} + \frac{1}{4}$

$36 = 4x - 16 + x - 3$

$36 = 5x - 19$

$55 = 5x$

$x = 11$

69.) $x = 11$

Chapter 12 SEQUENCES AND SERIES

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$

Find the sum of the series.

71) $\sum_{n=1}^{6}(n^2+7)$ ↙ not arithmetic

$= 8 + 11 + 16 + 23 + 32 + 43$
$= 133$

72) $\sum_{n=2}^{6}(10-4n)$

$n = 5$ $\quad S_5 = 5\left(\frac{2 + -14}{2}\right)$
$a_1 = 2$
$a_5 = -14$ $\quad S_5 = -30$

73) $\sum_{n=1}^{17} n$

$n = 17$
$a_1 = 1$
$a_{17} = 17$
$S_{17} = 17\left(\frac{1+17}{2}\right)$
$= 153$

133

71) (71)

72) -30

73) 153

Write a rule for the nth term of the arithmetic sequence.

$a_n = a_1 + (n-1)d$

74) 8, 5, 2, -1, -4, ...
$a_1 = 8$, $d = -3$
$a_n = 8 + (n-1)(-3)$
$a_n = 8 - 3n + 3$
$a_n = 11 - 3n$

75) $d = 7$, $a_8 = 54$
$54 = a_1 + 7(7)$
$54 = a_1 + 49$
$a_1 = 5$
$a_n = 5 + (n-1)7$
$a_n = 5 + 7n - 7$
$a_n = 7n - 2$

76) $a_4 = 27$, $a_{11} = 69$
$27 = a_1 + 3d$ $\quad 69 = a_1 + 10d$
$a_1 = 27 - 3d$ $\quad 69 = 27 - 3d + 10d$
$42 = 7d$
$d = 6$
$a_1 = 27 - 18$
$a_1 = 9$
$a_n = 9 + (n-1)6$
$a_n = 9 + 6n - 6$

74) $a_n = 11 - 3n$

75) ~~$a_n = 7n + 5$~~ $a_n = 7n - 2$

76) $a_n = 6n + 3$

Find the sum of the series.

77) $\sum_{i=1}^{15}(3+2i)$

$n = 15$
$a_1 = 5$ $\quad a_{15} = 33$
$S_{15} = 15\left(\frac{5+33}{2}\right)$
$= 285$

78) $\sum_{i=1}^{26}(25-3i)$

$n = 26$
$a_1 = 22$ $\quad a_{26} = -53$
$S_{26} = 26\left(\frac{22 + -53}{2}\right)$
$S_{26} = -403$

79) $\sum_{i=1}^{22}(6i-5)$

$n = 22$
$a_1 = 1$ $\quad a_{22} = 127$
$S_{22} = 22\left(\frac{1+127}{2}\right)$
$= 1408$

77) 285

78) -403

79) 1408

80) Joe buys a $600 computer on layaway by making a $200 down payment and then paying $25 per month. Write a rule for the total amount of money paid on the computer after *n* months.

80)

$a_1 = 200$
$d = 25$
$a_n = 200 + (n - 1)25$
$an = 175 + 25n$

Write a rule for the nth term of the geometric sequence.

$a_n = a_1 r^{n-1}$

81) 256, 64, 16, 4, 1, ...
$a_1 = 256$ $\quad r = \frac{1}{4}$
$a_n = 256\left(\frac{1}{4}\right)^{n-1}$

82) $r = 5$, $a_2 = 200$
$200 = a_1 5^1$
$a_1 = 40$
$a_n = 40(5)^{n-1}$

83) $a_3 = 16$, $a_5 = \frac{16}{9}$
$16 = a_1 r^2$ $\quad \frac{16}{9} = a_1 r^4$
$a_1 = \frac{16}{r^2}$ $\quad \frac{16}{9} = \frac{16r^4}{r^2}$
$16 = a_1\left(\frac{1}{3}\right)^2$ $\quad \frac{16}{9} = 16r^2$
$16 = a_1\left(\frac{1}{9}\right)$ $\quad \frac{1}{9} = r^2$
$a_1 = 144$ $\quad r = \frac{1}{3}$

81) $a_n = 256\left(\frac{1}{4}\right)^{n-1}$

82) $a_n = 40(5)^{n-1}$

83) $a_n = 144\left(\frac{1}{3}\right)^{n-1}$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $S = \frac{a_1}{1-r}$

Find the sum of the series, if it exists. If it does not exist, write "no limit exists."

84) $\sum_{i=1}^{9} 8(2)^{i-1}$ $n=9$, $a_1=8$, $r=2$

$S_9 = 8\left(\frac{1-2^9}{1-2}\right)$

$= 4088$

85) $\sum_{i=1}^{7} 40\left(\frac{1}{2}\right)^{i-1}$ $n=7$, $a_1=40$, $r=1/2$

$S_7 = 40\left(\frac{1-(1/2)^7}{1-1/2}\right)$

$= \frac{635}{8}$

86) $\sum_{i=1}^{\infty} 3\left(\frac{5}{8}\right)^{i-1}$ $a_1=3$, $r=5/8$

$S = \frac{3}{1-5/8} = 8$

84) 4088

85) $\frac{635}{8}$

86) 8

87) $\sum_{i=1}^{\infty} 7\left(-\frac{3}{4}\right)^{i-1}$ $a_1=7$, $r=-3/4$

$S = \frac{7}{1+3/4} = 4$

88) $\sum_{i=1}^{\infty} 4(1.25)^{i-1}$

89) $\sum_{i=1}^{\infty} \frac{2}{3}(-3)^{i-1}$

87) 4

88) No sum

89) No sum

Write the first five terms of the sequence.

90) $a_1 = 4,\ a_n = a_{n-1} + 9$

$a_2 = 4+9 = 13$

$a_3 = 13+9 = 22$

$a_4 = 22+9 = 31$

$a_5 = 31+9 = 40$

91) $a_1 = 8,\ a_n = 5a_{n-1}$

$a_2 = 5(8) = 40$

$a_3 = 5(40) = 200$

$a_4 = 5(200) = 1000$

$a_5 = 5(1000) = 5000$

92) $a_1 = 2,\ a_n = n \cdot a_{n-1}$

$a_2 = 2(2) = 4$

$a_3 = 3(4) = 12$

$a_4 = 4(12) = 48$

$a_5 = 5(48) = 240$

93) $a_1 = 4,\ a_2 = 7,\ a_n = a_{n-1} + a_{n-2}$

$a_3 = 7+4 = 11$

$a_4 = 11+7 = 18$

$a_5 = 18+11 = 29$

90) 4, 13, 22, 31, 40

91) 8, 40, 200, 1000, ~~5000~~ 5000

92) 2, 4, 12, 48, 240

93) 4, 7, 11, 18, 29

Write a recursive rule for the sequence.

94) 2, 12, 72, 432, ... $r = 6$

$a_1 = 2$

$a_n = 6 \cdot a_{n-1}$

95) 3, 10, 17, 24, ... $d = 7$

$a_1 = 3$

$a_n = a_{n-1} + 7$

94) $a_1 = 2,\ a_n = 6 \cdot a_{n-1}$

95) $a_1 = 3,\ a_n = a_{n-1} + 7$

Find the first three iterates of the function for the given initial value.

96) $f(x) = 3x - 7,\ x_0 = 4$

$f(4) = 12-7 = 5$

$f(5) = 15-7 = 8$

$f(8) = 24-7 = 17$

97) $f(x) = 8 - 5x,\ x_0 = 1$

$f(1) = 8-5 = 3$

$f(3) = 8-15 = -7$

$f(-7) = 8+35 = 43$

96) 5, 8, 17

97) 3, −7, 43

Chapter 10 COUNTING METHODS AND PROBABILITY

98) A briefcase lock has 3 rotating cylinders each containing 10 digits. How many numerical codes are possible?

10^3

98) 1000

99) Alan is playing the role of Oliver in his school's production of *Oliver Twist*. The wardrobe crew has presented Allan with 5 pairs of pants and 4 shirts that he can wear. How many possible costumes consisting of a pair of pants and a shirt does Allan have to choose from?

$5C_1 \cdot 4C_1 = 5 \cdot 4$

99) 20

100) A Mexican restaurant offers chicken, beef, or vegetarian fajitas wrapped with either corn or flour tortillas, and topped with either mild, medium or hot salsa. How many different choices of fajitas does a customer have?

$3 \cdot 2 \cdot 3$

100) 18

101) How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and no digit can be repeated? 8 9 8 7 6 5 4

101) 483,840

Determine whether each situation involves a permutation of combination. Then find the number of possibilities.

102) Seating 8 students in 8 seats in the front row of the school auditorium.

102) perm, $8!$ 40320

103) Checking out 3 library books from a list of 8 books for a research paper.

$8C_3$

103) 56

104) Electing 4 candidates to a municipal planning board from a field of 7 candidates.

$7C_4$

104) 35

105) The first, second and third place finishers in a race with 10 contestants.

$10P_3$

105) 720

Evaluate.

106) ${}_5P_3$ 107) ${}_6C_2$ 108) $6!$ 109) $\frac{10!}{5!5!}$

106) 60

107) 15

108) 720

109) 252

110) Find the number of distinguishable permutations in the following word:

a) PANAMA $\frac{6!}{3!}$ b) FACTORIAL $\frac{9!}{2!}$ c) MISSISSIPPI $\frac{11!}{4!4!2!}$

110a) 120

110b) 181440

110c) 34650

111) Find the number of possible 5-card hands that contain the cards specified. The cards are taken from a standard 52-card deck.

a) 4 kings and one other card $_4C_4 \cdot {}_{48}C_1$ 111a) 48

b) 5 hearts or 5 diamonds 111b) 2574

$_{13}C_5 + {}_{13}C_5$

113) Six representatives from a senior class of 350 students are to be chosen for the student council. In how many ways can these students be chosen to represent the senior class on the student council?

113) $_{350}C_6$

114) You have an equally likely chance of choosing any integer from 1 through 30. Find the probability of the given event.

a) An even number is chosen $\frac{15}{30}$

b) A multiple of 5 is chosen $\frac{6}{30} = \frac{1}{5}$

114a) 1/2

114b) 1/5

c) A factor of 60 is chosen 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, ~~60~~ $\frac{11}{30}$

d) A prime number is chosen 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

114c) 11/30

114d) 1/3

115) Let A and B be events such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{3}$. Find $P(A \text{ or } B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= \frac{2}{3} + \frac{1}{2} - \frac{1}{3}$

$= \frac{4}{6} + \frac{3}{6} - \frac{2}{6} = \frac{5}{6}$

115) 5/6

116) Let A and B be events such that P(A) = 0.32, P(B) = 0.48, and P(A and B) = 0.12. Find the indicated probability.

a) P(A or B) b) $P(\overline{A})$ c) $P(\overline{B})$

$.32 + .48 - .12$ $1 - .32$ $1 - .48$

$= .8 - .12$ $= .68$ $= .52$

$= .68$

116a) .68

116b) .68

161c) .52

117) A card is randomly selected from a standard deck of 52 cards. Find the probability of drawing the given card.

a) a red king b) a diamond or a 3 c) not a club

$\frac{2}{52} = \frac{1}{26}$ (13/52)+(4/52)-(1/52) $\frac{39}{52} = \frac{3}{4}$

=4/13

P(red and king)

= P(red) + P(king) - P(red or king)

26/52 4/52 28/52

117a) 1/26

117b) 4/13

117c) 3/4

118) Two six-sided dice are rolled. Find the probability of the given event.

a) The sum is not 7 b) The sum is less than 8 or greater than 11

P(sum not 7) = 1 - P(sum 7)

$= 1 - \frac{6}{36}$

$= 30/36 = 5/6$

P(sum < 8) + P(sum > 11)

$\frac{21}{36} + \frac{1}{36} = \frac{22}{36} = \frac{11}{18}$

118a.) 5/6

118b.) 11/18

Find the odds in favor of an event, given the probability of the event.

119) $\frac{3}{7}$ 120) $\frac{4}{5}$ 121) $\frac{1}{15}$

119) 3:4

120) 4:1

121) 1:14

Find the probability of an event occurring, given the odds of the event.

122) 10:1 123) 4:9 124) 8:3

Denom

4 + 9 = 13

$\frac{4}{13}$ $\frac{9}{13}$

122) $\frac{10}{11}$

123) $\frac{4}{13}$

124) $\frac{8}{11}$

125) A die is rolled twice. Find the probability. Independent events

a) P(5, then 6) $= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

b) P(4, then not 6) $= \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{36}$

125 a) 1/36

b) 5/36

126) There are 3 nickels, 3 dimes and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

a) P(nickel, then dime, then quarter) if no replacement occurs.

126 a) 1/22

$$\frac{3C_1}{11C_1}\cdot\frac{3C_1}{10C_1}\cdot\frac{5C_1}{9C_1}=\frac{3\cdot3\cdot5}{11\cdot10\cdot9}=\frac{45}{990}=\frac{1}{22}$$

b) P(3 dimes) if replacement occurs.

b) 27/1331

$$\frac{3C_1}{11C_1}\cdot\frac{3C_1}{11C_1}\cdot\frac{3C_1}{11C_1}=\frac{3^3}{11^3}=\frac{27}{1331}$$

c) P(nickel, then 2 quarters) if replacement occurs.

c) 75/1331

$$\frac{3C_1}{11C_1}\cdot\frac{5C_1}{11C_1}\cdot\frac{5C_1}{11C_1}=\frac{3\cdot5\cdot5}{11^3}=\frac{75}{1331}$$

d) P(3 quarters) if no replacement occurs.

d) 2/33

$$\frac{5C_1}{11C_1}\cdot\frac{4C_1}{10C_1}\cdot\frac{3C_1}{9C_1}=\frac{5\cdot4\cdot3}{11\cdot10\cdot9}=\frac{60}{990}=\frac{2}{33}$$

127) Serena is creating a painting. She wants to use 2 more colors. She chooses randomly from 6 shades of red, 10 shades of green, 4 shades of yellow, 4 shades of purple and 6 shades of blue. What is the probability that she chooses 2 shades of green?

127) 3/29

$$\frac{10C_2}{30C_2}=\frac{45}{435}=\frac{3}{29}$$

128) Becky's mother is shopping at the bakery. The owner offers Becky a cookie from a jar containing 22 chocolate chip cookies, 18 sugar cookies and 15 oatmeal cookies. Without looking, Becky selects one, drops it back in, and then randomly selects another. What is the probability that neither selection was a chocolate chip cookie?

128) 9/25

$$\frac{33C_1}{55C_1}\cdot\frac{33C_1}{55C_1}=\frac{1089}{3025}=\frac{9}{25}$$

129) A die is rolled. Find each probability.

a) P(5 or 6)

$$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$$

b) P(at least a 3)

$$\frac{4}{6}=\frac{2}{3}$$

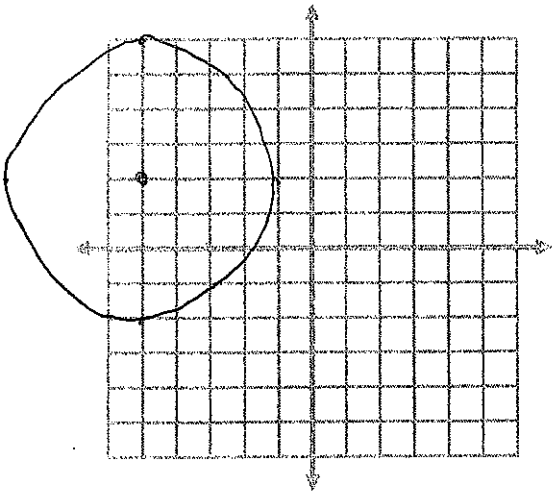
129 a) 1/3

b) 2/3

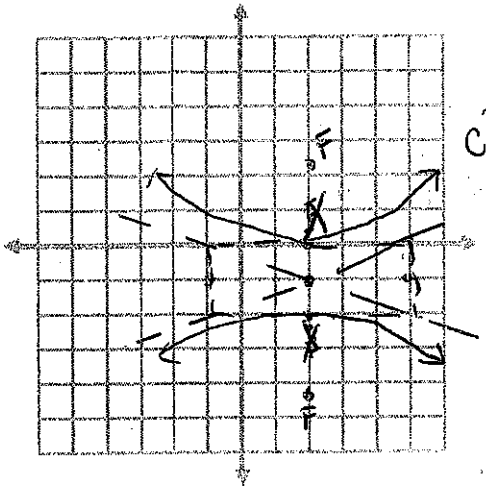
Chapter 9 Conics

Graph the following equations.

130. $(x+5)^2+(y-2)^2=16$



131. $(y+1)^2-\frac{(x-2)^2}{9}=1$



$c^2=a^2+b^2$

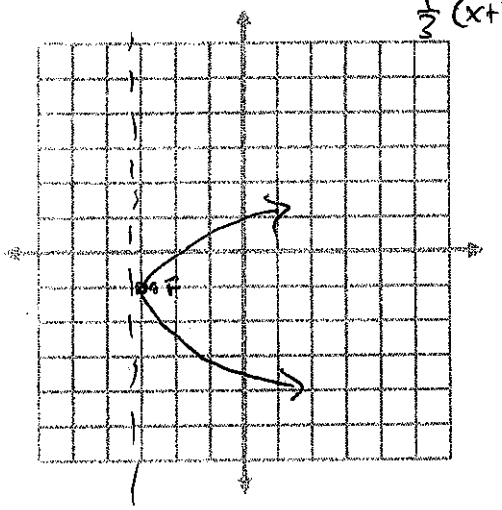
$c^2=1+9$

$c^2=10$

$c=\sqrt{10}$

132. $x=3(y+1)^2-3$ $(x+3)=3(y+1)^2$

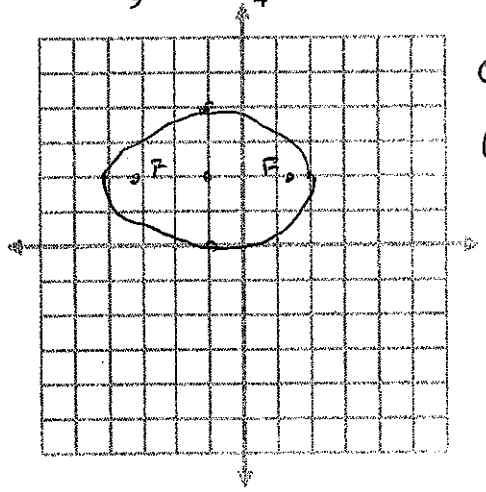
$\frac{1}{3}(x+3)=(y+1)^2$



$\frac{1}{3}=4p$

$p=\frac{1}{12}$

133. $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$



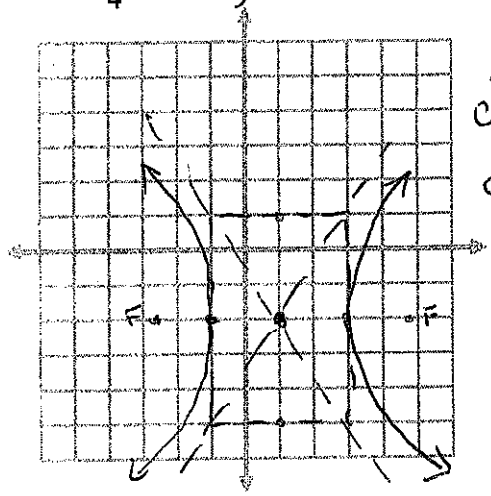
$c^2=a^2-b^2$

$c^2=9-4$

$c^2=5$

$c=\sqrt{5}$

134. $\frac{(x-1)^2}{4}-\frac{(y+2)^2}{9}=1$



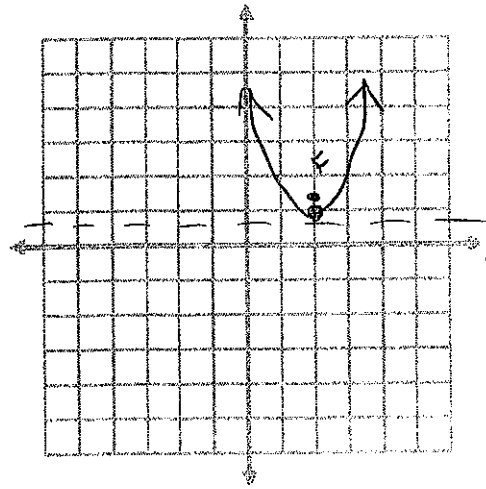
$c^2=a^2+b^2$

$c^2=9+4$

$c^2=13$

$c=\sqrt{13}$

135. $y=(x-2)^2+1$ $(x-2)^2=y-1$



$4p=1$

$p=\frac{1}{4}$

Vertex: $(2,1)$

Focus: $(2,\frac{5}{4})$

directrix: $y=\frac{3}{4}$

Write an equation given the following information.

136. Circle Center (-2, 4) and a point on the circle (-7, 5)

$(x+2)^2 + (y-4)^2 = r^2$

$25 + 1 = r^2$

$r^2 = 26$

136. $(x+2)^2 + (y-4)^2 = 26$

137. Parabola Vertex at (-3, -4) Focus (-3, 3¾)

$p = \frac{31}{4}$

$(x+3)^2 = 4p(y+4)$

$(x+3)^2 = 31(y+4)$

137. $(x+3)^2 = 31(y+4)$

138. Parabola Vertex at (1, 3) directrix x = 7/8

$(y-3)^2 = 4p(x-1) \quad p = \frac{1}{8}$

$(y-3)^2 = \frac{1}{2}(x-1)$

138. $(y-3)^2 = \frac{1}{2}(x-1)$

139. Ellipse Vertices (4, 2) (4, -8) Co-vertices (1, -3) (7, -3) Center (4, -3)

$\frac{(x-4)^2}{9} + \frac{(y+3)^2}{25} = 1$

139. $\frac{(x-4)^2}{9} + \frac{(y+3)^2}{25} = 1$

140. Hyperbola Vertices (-4, 3) (-4, 7) Foci (-4, 1) (-4, 9) Center (-4, 5)

$a = 2, a^2 = 4$

$c = 4, c^2 = 16$

$c^2 = a^2 + b^2$

$16 = 4 + b^2$

$b^2 = 12$

140. $\frac{(y-5)^2}{4} - \frac{(x+4)^2}{12} = 1$

Identify the vertices, foci, co-vertices, and directrix from the graph – if they exist.

141. $(y+2)^2 = \frac{1}{2}(x+3)$

$4p = \frac{1}{2}$

$p = \frac{1}{8}$

141. V $(-3, -2)$

F $(-\frac{23}{8}, -2)$

CV ———

d $x = -\frac{25}{8}$

142. $\frac{(y+5)^2}{3} - \frac{(x+4)^2}{9} = 1$

center $(-4, -5)$

$a = \sqrt{3} \quad b = 3$

$c^2 = 3 + 9 = 12$

$c = \sqrt{12} = 2\sqrt{3}$

V: $(-4, -5 \pm \sqrt{3})$

F: $(-4, -5 \pm 2\sqrt{3})$

142. V ———

F ———

CV ———

d ———

143. $\frac{(x-4)^2}{36}+\frac{(y+2)^2}{7}=1$

Center $(4,-2)$

V: $(-2,-2)$ $(10,-2)$

CV: $(4,-2\pm\sqrt{7})$

F: $(4\pm\sqrt{29},-2)$

$c^2=36-7$

$c^2=29$ $c=\sqrt{29}$

144. $(x+4)^2+(y-2)^2=25$

Center $(-4,2)$

$r=5$

143. V _____

F _____

CV _____

d —

144. V —

F —

CV —

d —

Without graphing, identify the conic from the equation.

145. $\frac{(x+1)^2}{16}-\frac{(y-2)^2}{7}=1$

146. $x=(y-2)^2+3$

145. hyperbola

146. parabola

147. $\frac{(x-3)^2}{5}+(y+2)^2=1$

148. $(x-2)^2+(y+3)^2=25$

147. ellipse

148. circle

$\frac{(x-3)^2}{5}+\frac{5(y+2)^2}{5}=\frac{5}{5}$